MULTIPLE REPRESENTATIONS
CHAPTER 4  Multiple Representations

This chapter builds on the work you did in the previous two chapters. Chapter 4 contains only one section, as it focuses solely on the connections between the four representations of data: patterns, tables, graphs, and equations (also referred to as “rules”).

In this chapter, you will learn:

- How to change any representation of data (such as a pattern, table, graph or rule) to any of the other representations.
- How to use the connections between patterns, tables, graphs, and rules to solve problems.

Guiding Questions

Think about these questions throughout this chapter:

What is the connection?

Is there a pattern?

How does the pattern grow?

In how many different ways can it be represented?

How does the pattern show up in the table, graph, and rule?

Chapter Outline

Section 4.1  You will shift between different representations of linear patterns, using the web diagram shown at left. By finding connections between each representation, you and your team will find ways to change from one representation to each of the other three representations.
### Chapter 4 Teacher Guide

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**Total:** 9 days plus optional time for Closure and Assessment
Overview of the Chapter

This chapter is designed to accomplish several objectives:

- To have students find connections between the four representations: graphs, tables, patterns, and rules.
- To challenge teams with difficult problems that require them to think deeply about the relationships between the different representations.
- To begin a focus on writing equations from word problems, which is continued in Chapter 5.
- To introduce students to solving systems of equations where both equations are in $y = mx + b$ form.

At the beginning of Section 4.1, teams are challenged with a difficult, non-linear pattern for which they generate multiple representations (graph, table, tile pattern, and rule) and for which they predict the shape and area of the 100th figure. Then they focus on finding connections between different representations in the web (shown at right).

From this non-linear challenge, the chapter returns to linear patterns to help students focus on the similarities between representations of a pattern. For example, when students notice that a geometric tile pattern grows by two tiles from figure to figure, they will make a connection to how that growth is represented in the rule (the coefficient of $x$), in the table (each $y$-value is 2 more than the one before), and the graph (where points “step up” or “step-down” each time, shown with growth triangles). By the end of Section 4.1, students should be able to change any representation to any other for linear patterns.

Note that the web will help students keep track of which connections they have developed throughout the chapter. From Chapter 3, students know how to make the following connections: rule → table, graph → table, pattern → table, table → rule, and table → graph.

Connections that are explored in Section 4.1 are shown in the diagram at right. The dashed lines represent connections that are explored, but are not complete.
Common Core State Standards for Mathematical Practice

As students move into Chapter 4, they should be starting to use some of the Mathematical Practices with more regularity. It should only take a gentle reminder from you to **attend to precision** in their communication with each other. They should be more comfortable **constructing viable arguments and critiquing the reasoning of others** as you encourage discourse during class. Students should now want to **make sense of the problems** that you ask them to attempt and they should be starting to show more and more **perseverance in solving them**.

In this chapter, you will guide students to **look for and make use of the mathematical structure** a linear equation as they use **repeated reasoning** to make connections and build understanding.

**Where Is This Going?**

Students will study functions with tables, graphs and rules again in Chapter 8 (exponential growth and functions/relations). The tools and connections that they develop in this chapter will continue to be necessary throughout this course, but particularly in Chapter 5 (systems of equations), and Chapter 7 \( y = mx + b \).
Suggested Assessment Plan for Chapter 4

For complete discussion and recommendations about assessment strategies and grading, refer to the Assessment section of the Teacher Edition.

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<td>Support quality teamwork as teams begin to develop their understanding of linear patterns.</td>
<td>Choose parts of this team challenge for teams to present based on your observations of their work. Be sure presentations highlight multiple methods.</td>
<td>This challenging problem allows students to showcase their teams’ ability to find connections between multiple representations of a quadratic function.</td>
</tr>
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**Ideas for Team Test**

On a Chapter 4 team test, it is appropriate to include more challenging problems about multiple representations. Be sure to look for problems in which there are mathematical ideas rich enough to inspire team discussions and to include problems involving previous material. See the online Assessment Bank for examples and further ideas.

**Ideas for Individual Test**

This chapter establishes the Multiple Representations of Patterns focus of this course. When designing individual assessments, it is important to allow room for students to see a problem and its solution in multiple ways. Be sure to check that your test questions do not force students away from the representations with which they may feel most comfortable at this point.

It is strongly recommended that more than half of each test be made up of material from previous chapters. Along with any previous material, it is now appropriate to test students’:

- Ability to solve simple equations as in problems 4-9, 4-17, 4-27, 4-39, and 4-51.
- Ability to graph linear or quadratic equations by first making a table as in problems 4-7, 4-44, and 4-50.
- Understanding of points as solutions as in problems 4-44 and 4-50.
- Ability to use integer values of $m$ and $b$ to graph lines given in $y = mx + b$ form without first making a table as in problems 4-59 and 4-67.
- Mastery of Checkpoint 4: Area and perimeter of circles and complex figures as in problems 4-21, 4-26, 4-46, 4-62, and 4-71.
Lesson 4.1.1  What is the connection?

Finding Connections Between Representations

Lesson Objective: Students will discover connections between all of the representations of a pattern: a graph, a table, a geometric presentation, and an equation. Students will also look at different ways to represent the connections.

Mathematical Practices: As students investigate the connections between tile patterns, tables, graphs, and rules, they are reasoning abstractly and quantitatively. When they write a rule for their tile pattern, they are modeling with mathematics.

Length of Activity: Two days (approximately 90 minutes)

Core Problems: Problem 4-1

Materials: Butcher paper or poster paper (graph poster paper may be helpful), one per team
Markers or colored pencils
Lesson 4.1.1A Resource Page (“Tile Pattern Team Challenge”), 5 pages in all, enough copies so that each team gets two copies of their tile pattern (see note in “Suggested Lesson Activity” below for more details)
Lesson 4.1.1B Resource Page (“Gallery Walk”), for board display (optional)
Lesson 4.1.1C Resource Page (“Team Roles”), one per team and one for board display (optional)

Lesson Overview: This activity is a two-day team challenge in which students use the tools they have developed in Chapters 1 through 3 to represent a pattern in multiple ways. Each study team will analyze a non-linear tile pattern and will generate the other representations of the figure number and the number of tiles it contains (graph, \(x \rightarrow y\) table, drawing, rule). Then students will find as many connections as possible between the representations. Each team will present the connections they found to the class on Day 2.

Suggested Lesson Activity: Begin Day 1 by introducing the task. Let students know that each team will have a different tile pattern to work on and that teams will be responsible for teaching the class about their patterns. Discuss the idea of “multiple representations,” eliciting from students the different ways they have represented patterns in the first three chapters. The lesson introduction before problem 4-1 states these representations formally.

You can start problem 4-1 as a Teammates Consult. Make sure that everyone understands the task before picking up their pencils and starting the task. If students will be working from the textbook, assign each team
a pattern from problem 4-1. Depending on the size of your class, you may need to give some teams the same pattern. Or distribute the Lesson 4.1.1A Resource Page (“Tile Pattern Team Challenge”), which contains the task instructions so that students do not need their books on their desks. The Lesson 4.1.1A Resource Page includes five pages in all, each with a different tile pattern. Each team should receive two copies of the resource page for their pattern.

Students should do their work on graph paper, which makes it easier to draw the tile patterns clearly. Teams will take the remainder of the class period to complete the task. Some teams may begin their poster. Remind students as you circulate that although each student will need to turn in the pattern analysis individually, students should be working in their teams and discussing each question together before moving on. This can be done as a Huddle by bringing one person from each team up to the front of the class to share the information.

Emphasize the importance of showing connections between representations. One way to do this is to ask students to describe the patterns they notice, and then to ask them how they can find the same pattern in a different representation. For example, if students see that tiles are configured so that one portion is a rectangle that has dimensions “Figure # + 1” by “Figure # + 2,” encourage them to think about how that will appear in Figure 100, or how that pattern might help them write a rule. One way students could show how they see their pattern related to its figure number is using a diagram like the one shown below.

In their homework on the first day of the activity (problem 4-2), students are asked to extend each of the tile patterns given in problem 4-1. The purpose of this task is for students to become familiar with the patterns they will see presented the next day. If they are already familiar with the patterns, they will be able to ask questions about the presenting team’s generalization of the pattern more easily. Remind students that they do not have to complete the problem 4-1 questions for each pattern; they only need to create Figures 0, 4, and 5 for each pattern.

On the second day, start class by reviewing the expectations for the posters and presentations, and outlining the time students have remaining. Remind students that their final product will include each team member’s individual analysis of the pattern as well as a team poster summarizing the connections between representations. Allow about five minutes per team presentation.
Option 1: Team Poster Presentations

Between presentations, while the presenting team puts its poster up on the wall, you may want to ask students to review the work they did in homework on the pattern about to be presented. This familiarity with the starting pattern will facilitate students’ asking questions of the presenting team.

As teams present their posters, ask the audience to reflect about the focus questions for the day:

How does this team see the pattern?

How is this team showing growth in each representation?

What are the connections? How are they showing them?

Teachers may want to discuss these questions with the whole class after each presentation, or after all of the presentations are completed.

Questions that could be used to spark the discussion include: “How did this team help you see their pattern?”, “How did they highlight their connections?”, “How are the different representations connected?”, and “What are the advantages of each way of showing those connections?”

Option 2: Gallery Walk

An alternative to using presentations to close this activity is to set up what is called a “Gallery Walk” with the posters that teams created. In a Gallery Walk, each team puts their poster on the wall. Teams then split up and look at posters created by other students and create some kind of written feedback for the creators. This can be done with sticky notes.

The Lesson 4.1.1B Resource Page, entitled “Gallery Walk,” provides a model to do this. It includes the following directions, which you can display on the board during the Gallery Walk.

**Gallery Walk Directions**

You will now have an opportunity to examine the posters created by the other teams. As you circulate, give written comments about the following:

- Does the poster describe growth in the table? In the pattern? In the graph? In the rule?
- Does the poster describe the pattern?
- Which connections does it show? For example, does the poster show connections between the table and graph? Between the pattern and rule? Pattern and graph?
- What suggestions do you have for this team? (Be serious, positive, and helpful!)
- What do you like about this team’s work?
Universal Access: **Academic Literacy and Language Support:** If your class has a large number of ELL and/or students needing additional support, it is recommended that you use Option #1 Team Presentations as the follow-up for problem 4-1. The Gallery Walk can be incorporated into later chapters. The team presentations allow the teacher to facilitate the discussion and guide the class using the focus questions for this lesson. Support students with language development by having them use vocabulary from the word wall as they explain their work. An additional support would be to have students Pair Share portions of the presentations before having them come up in the front of the class as a team.

The non-linear rules in problem 4-1 are challenging. Students may need some guidance with finding the rules from the table. If they cannot find the rule for the pattern, encourage them to work with the other multiple representations - pattern, table, and graph. As students work in this section continue to have them focus on the relationship $x$ as the *figure #* and $y$ as the *total number of tiles in a specific figure*. This connection will help students in Chapter 7 when the concept of slope is introduced.

Team Strategies: This activity is an ideal time for students to use team roles to facilitate the completion of a task. The 4.1.1C Resource Page (“Team Roles”) provides prompts for team roles. This resource page is slightly different than the General Team Roles Resource Page from Chapter 1 because it is directed at the specific tasks for this lesson. Because of the length of the activity, it is strongly encouraged that you to distribute the copies of the “Team Roles” resource page as well as display it on the board, so that students can readily refer back to it as they work.

If you use team roles with this task, highlight the importance of using the roles to help students have good mathematical conversations as they complete the task. Show the role descriptions on the board, and then have the Resource Managers pick up the “Tile Pattern Team Challenge” resource pages.

Prompt Facilitators to make sure the roles on the board and the task statements on the resource pages are read aloud before the team begins work. The Recorder/Reporter is in charge of designing the poster and assigning a task for each person to do on the poster. The Task Manager watches the time and keeps everyone on task.

If you will be closing this activity with a Gallery Walk (see the “Option 2: Gallery Walk” notes in the “Closure” section above), it might be a good idea to assign one of the four roles (such as the Recorder/Reporter) the responsibility of staying with the team poster while the other team members visit the posters of other teams.

The team roles for this activity are reprinted below for your convenience:
Resource Manager:
- Get materials for your team.
- Make sure that all questions are team questions.
  “What team question can we ask the teacher?”
  “Are we sure that no one here can answer the question?”
- Do not let your team stay stuck!

Facilitator:
- Make sure your team understands the entire task before you begin.
  “Who wants to read? Does everyone understand what we are asked to do?”
  “What is the connection? How will it show in the graph? How will it show in the \( x \rightarrow y \) table?”
- Keep your team together. Make sure everyone’s ideas are heard.
  “Does anyone see it a different way?”

Recorder/Reporter:
- Help your team organize a poster with all of your results. Your poster needs to show everyone’s ideas and be well-organized. Use color, arrows, and other math tools to communicate your team’s mathematics, reasoning, and connections.
  “How can we show the growth?”
  “How can we show that connection?”

Task Manager:
- Be sure that your team is accomplishing the task effectively and efficiently. Keep track of the time and tell the team when it is time to move forward to the next part of the task. Make sure that all talking is within your team and is helping you to accomplish the task. Eliminate side conversations.
  “Are we ready to move on?”
  “How can we divide the work most efficiently?”
  “We need to finish this part in 5 minutes, so we have time for…”

Homework:
Day 1: Problem 4-2 through 4-6
Day 2: Problems 4-7 through 4-11

For presentations on the second day to be more meaningful, problem 4-2 asks students to extend each of the patterns in problem 4-1 for homework. Remind students that they do not have to complete the problem 4-1 questions for each pattern; they only need to create Figures 0, 4, and 5 for each pattern.

Notes to Self:
4.1.1 What is the connection?

Finding Connections Between Representations

In Chapter 3, you studied different ways to represent patterns. You organized information into tables, graphed information about patterns, and learned how to find the rules that govern specific patterns.

Starting today and continuing throughout this chapter, you will find connections between different representations of the same pattern. You will also explore each representation in further depth. This work will develop efficient ways to go from one representation to another. By the end of this chapter, you will have a deeper understanding of many of the most powerful tools of algebra.

4-1. TILE PATTERN TEAM CHALLENGE

Your teacher will assign your team a tile pattern (one of the patterns labeled (a) through (e) on the next page). Your team’s task is to create a poster showing every way you can represent your pattern and highlighting all of the connections between the representations that you can find. Finding and showing the connections are the most important parts of this activity. Clearly presenting the connections between representations on your poster will help you convince your classmates that your description of the pattern makes sense. [Rules vary. See the “Suggested Lesson Activity” for an example; however, the rules should be able to simplify to these expressions: a: $n^2 + 5n + 6$, b: $n^2 + 4n + 6$, c: $n^2 + 4n + 4$, d: $n^2 + 6n + 6$, e: $n^2 + 4n + 4$.]

Pattern Analysis:

- Extend the pattern by drawing Figures 0, 4, and 5. Then describe Figure 100. Give as much information as you can about Figure 100. What will it look like? How will the tiles be arranged? How many tiles will it have?
- Find the number of tiles in each figure. Record your data in a table and on a graph.
- Generalize the pattern by writing a rule that will give the number of tiles in any figure in the pattern. Show how you got your answer.
- Demonstrate how the pattern grows. Use color, arrows, labels, and other math tools to help you show and explain. Show growth in each representation.
- What connections do you see between the different representations (figures, $x\rightarrow y$ table, and graph)? How can you show these connections?

Problem continues on next page. →
4-1. *Problem continued from previous page.*

**Presenting the Connections:**

As a team, organize your work into a large poster that clearly shows each representation of your pattern, as well as a description of Figure 100. When your team presents your poster to the class, you will need to support each statement with a reason from your observations. Each team member must explain something mathematical as part of your presentation.

a.  

![Figure 1](image1.png)  
**Figure 1**  

![Figure 2](image2.png)  
**Figure 2**  

![Figure 3](image3.png)  
**Figure 3**

b.  

![Figure 1](image4.png)  
**Figure 1**  

![Figure 2](image5.png)  
**Figure 2**  

![Figure 3](image6.png)  
**Figure 3**

c.  

![Figure 1](image7.png)  
**Figure 1**  

![Figure 2](image8.png)  
**Figure 2**  

![Figure 3](image9.png)  
**Figure 3**

d.  

![Figure 1](image10.png)  
**Figure 1**  

![Figure 2](image11.png)  
**Figure 2**  

![Figure 3](image12.png)  
**Figure 3**

e.  

![Figure 1](image13.png)  
**Figure 1**  

![Figure 2](image14.png)  
**Figure 2**  

![Figure 3](image15.png)  
**Figure 3**

4-2. For each tile pattern in problem 4-1, draw Figures 0, 4, and 5 on graph paper. If it helps, copy Figures 1, 2, and 3 onto your paper. [See figures in the selected answers.]
4-3. On graph paper, draw Figure 0 and Figure 4 for the pattern at right.

a. Represent the number of tiles in each figure in an \(x \rightarrow y\) table. Let \(x\) be the figure number and \(y\) be the total number of tiles. [See table below right.]

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
</tr>
</tbody>
</table>

b. Use the table to graph the pattern. [See graph below right.]

c. Without drawing Figure 5, predict where its point would lie on the graph. Justify your prediction. [The point would lie at \((5, 19)\), because the rule is \(y = 3x + 4\) or because it is possible to predict on the graph.]

4-4. Evaluate the expressions below for the given values.

a. \(3(2x + 1)\) for \(x = -8\) \([-45]\)  
b. \(\frac{x-6}{4} - 1\) for \(x = -14\) \([-6]\)  
c. \(-2m^2 + 10\) for \(m = -6\) \([-62]\)  
d. \(k \cdot k + k \cdot k + k\) for \(k = 9\) \([9]\)

4-5. Copy and simplify the following expressions by combining like terms.

a. \(x + 3x - 3 + 2x^2 + 8 - 5x\) \([2x^2 - x + 5]\)  
b. \(2x + 4y^2 - 6y^2 - 9 + 1 - x + 3x\) \([-2y^2 + 4x - 8]\)  
c. \(2x^2 + 30y - 3y^2 + 4xy - 14 - x\) \([2x^2 + 30y - 3y^2 + 4xy - 14 - x]\)  
d. \(20 + 3xy - 3xy + y^2 + 10 - y^2\) \([30]\)

4-6. Use the Distributive Property to rewrite each expression.

a. \(3(2x - 7)\) \([6x - 21]\)  
b. \(-2(x - 7) + 5x\) \([3x + 14]\)  
c. \(5x + 10\) \([5(x + 2)]\)  
d. \(8x + 12y\) \([4(2x + 3y)]\)
4-7. Make an $x \rightarrow y$ table for the rule $y = x^2 - 2x$.

a. Plot and connect the points on a complete graph. [The parabola should pass through the points $(0, 0)$ and $(2, 0)$ and have a vertex at $(1, -1)$.]

b. Does your graph look like a full parabola? If not, add more points to your table and graph to complete the picture.

4-8. THE GAME SHOW

Susan had an incredible streak of good fortune as a guest on an exciting game show called “The Math Is Right.” She amassed winnings of $12,500, a sports car, two round-trip airline tickets, and five pieces of furniture.

In an amazing finish, Susan then landed on a “Double Your Prize” square and answered the corresponding math question correctly. She instantly became the show’s biggest winner ever, earning twice the amounts of all her previous prizes.

A week later, $25,000, a sports car, four round-trip airline tickets, and five pieces of furniture arrived at her house. Susan felt cheated. What was wrong? [She should have received two sports cars and ten pieces of furniture.]

4-9. Write the equation represented by the diagram at right.

\[1 - 3 - (-2x) = -x - 3 - (x + 2)\]

a. Simplify as much as possible and then solve for $x$. \[x = -0.75\]

b. Check your solution.

4-10. Copy and simplify the following expressions by combining like terms.

\[
\begin{align*}
&\text{a. } y + 3x - 3 + 2x^2 + 8x - 5y \\
&\quad \quad \quad \quad \quad \text{[ } 2x^2 + 11x - 4y - 3 \text{ ]} \\
&\text{b. } 2x + 4x^2 - 6x^2 - 9 + 1 - x - 3x \\
&\quad \quad \quad \quad \quad \text{[ } -2x^2 - 2x - 8 \text{ ]} \\
&\text{c. } 14 + 3y^2 + 30y - 3y^2 - 14y - 14 - 16y \\
&\quad \quad \quad \quad \quad \text{[ } 0 \text{ ]} \\
&\text{d. } -10x + 13y - 6x + 5y^2 + 10y - 5y^2 \\
&\quad \quad \quad \quad \quad \text{[ } -16x + 23y \text{ ]}
\end{align*}
\]

4-11. Use your pattern-finding techniques to fill in the missing entries for the table below. Then find a rule for the pattern. \[y = x^2 + 1\]

<table>
<thead>
<tr>
<th>IN (x)</th>
<th>4</th>
<th>8</th>
<th>3</th>
<th>-2</th>
<th>-6</th>
<th>0</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>OUT (y)</td>
<td>17</td>
<td>65</td>
<td>10</td>
<td>5</td>
<td>37</td>
<td>1</td>
<td>26</td>
<td>50</td>
</tr>
</tbody>
</table>
Lesson 4.1.2  How does it grow?

Seeing Growth in Different Representations

Lesson Objective: Students will write linear algebraic rules relating the figure number of a geometric pattern and its number of tiles. They will identify connections between the growth of a pattern and its linear equation.

Mathematical Practices: Students continue reasoning abstractly and quantitatively while working tile patterns. Today they make the connections more explicit, looking for and making use of the structure of a linear equation.

Length of Activity: One day (approximately 45 minutes)

Core Problems: Problems 4-12 through 4-15
Note: If students do not complete problem 4-15 in this lesson, have them start with problem 4-15 at the beginning of Lesson 4.1.3.

Materials: Lesson 4.1.2 Resource Page ("Pattern Analysis"), one per student
Poster paper for Representations of Patterns Web
Markers or colored pencils, four colors per team
General Team Roles Resource Page (optional)

Suggested Lesson Activity: Begin this lesson with a class discussion to summarize where the class is in its investigation of multiple representations of patterns. Ask students to name the representations they have worked with so far. This could be done with Reciprocal Teaching. Expect to hear students mention tile patterns, $x \rightarrow y$ tables, graphs, and equations (rules). Start by showing the names of the representations in the formation shown above on the board.

Then explain that in this chapter, students will be connecting all of the different representations and finding efficient ways to move from one representation to another. As you discuss this, fill in the arrows between the different representations to show what you mean. Explain using phrases such as, “We will find efficient ways to create a rule from a tile pattern, without creating an $x \rightarrow y$ table first” or “We will find ways to find an equation just by looking at a graph.” Continue until your diagram looks like the one at right.
Emphasize the reversibility of these processes as you describe them, noting that the arrowheads connecting representations are aimed in both directions. Point out that as students work with this lesson and upcoming lessons, they should be aware of which new connections they are making or which existing connections they are strengthening.

Then have a student read the lesson introduction, including the focus questions for the lesson. Start teams on problems 4-12 and 4-13, in which students are given three tile patterns and are asked to make predictions about each pattern. They will first draw the figures that immediately follow the ones provided and will also work backward to find the figure preceding the given patterns. The goal is for students to focus on how the tile arrangement is growing as new figures are added to the sequence. To save time, distribute the Lesson 4.1.2 Resource Page so students do not have to recopy the tile patterns and are not tempted to write in the text. Note that the final pattern on the resource page will not be needed until students get to problem 4-15.

In this lesson and in Lesson 4.1.3, students should use color to highlight growth. For example, students might color the tiles in problem 4-12 as shown below to represent which tiles were added to the previous figure.

There are many ways to see the growth; this example is not the only valid way to see tiles being added. Problem 4-13 asks students to use a single color to shade in tiles that are added to each new figure and to switch colors when they start to work on a new pattern. Students then use these patterns of growth to predict the total number and arrangement of tiles for Figure 100. If the teams are struggling or if they have an interesting strategy, you could use a Swapmeet to help disseminate ideas at any time.

The color-coding is important, as students will use color to match up the different representations of a single pattern over the course of Lessons 4.1.2 and 4.1.3. If colored pencils or markers are unavailable, or if it is impossible to expect students to retrieve the same four colors in Lesson 4.1.3, have students use a single color for all of the patterns and instead clearly label the $x \rightarrow y$ tables and graphs that they create in Lesson 4.1.3 with the tile-pattern number. This is not ideal, however; color visually links the different representations of a single pattern and helps students who are visual learners. An alternative to using color (especially if you have students with difficulty seeing different colors) is to have them use different types of shading instead.
Part (d) of problem 4-12 is left open so that students can apply a variety of strategies to draw their conclusions. Students will likely use both numerical and geometric approaches. Teams are encouraged to find as many ways as possible to describe and justify their observations.

When most teams have completed problems 4-12 and 4-13, bring the class together for a brief discussion of how the patterns change as the figure number increases. You may want to ask one team to demonstrate how they used shading to show growth in one of the patterns, and also ask a different team to present their reasoning for the total number of tiles and configuration of Figure 100 for the same pattern.

When you think that most students understand that they are looking for growth between figures, let teams continue work on problem 4-14, and if time allows, problem 4-15. Problem 4-14 asks students to connect the patterns of growth in the geometric representations to the numbers in an equation. If students do not complete problem 4-15 during this lesson, have them start Lesson 4.1.3 by analyzing the tile pattern in this problem.

Notice that all three patterns presented in problems 4-12 and 4-13 have a growth factor of 4. (“Growth factor” is the number of tiles the pattern increases each time you move from one figure to the next in the sequence. This term is introduced in problem 4-15.) This is intended to help students see a clear connection between patterns that grow by four tiles and $4x$ in the rule. In problem 4-15, students are presented with a pattern that grows by 2. As they compare Tile Patterns #3 and #4, they have the opportunity to see that each one has 3 tiles in Figure 0, the starting figure. This appears as the “added number” in the rule, although some teams may not see this at this point.

Note: Do not expect thorough understanding or formalization of $y = mx + b$ at this point. Ideas of growth and starting value represented as $m$ and $b$ will be developed throughout this chapter and again more thoroughly in Chapter 7. The word “$y$-intercept is introduced in Lesson 4.1.6, and the word “slope” is introduced in Chapter 7.

Closure:
(5 to 10 minutes)

Create a Learning Log entry as a whole class. Have a Representations of Patterns Web with no connections prepared on poster paper. Ask students what connections were worked on in class today (pattern $\rightarrow$ rule) and draw the appropriate arrow on the class poster. Then lead a discussion recalling all of the connections explored so far (in Chapter 3 and in Lesson 4.1.1) and add lines or arrows to represent these connections. Note that at this point, some of the connections are appropriate as arrows (the ones in which you can clearly get from one representation to another, such as table $\rightarrow$ graph), and some are appropriate as dashed lines to show that students have started a connection. At this point, your web diagram should look like the one at right.
Note: While students did study how to find rules from tables in Chapter 3, the arrow from table to rule is dashed at this point to indicate that this idea will be further developed (using \( y = mx + b \)) in this chapter.

Remind students to put their work from this lesson in a secure place because they will need it for the next lesson’s activity.

**Universal Access:**

**Academic Literacy and Language Support:** Encourage students to use one color to shade in the growth of each pattern. Refer to the Suggested Lesson Activity notes on the reasoning behind using a single color to represent the growth for each pattern.

To enable students to focus on drawing the new figures in the pattern, provide Resource Page 4.1.2 for problem 4-12. This way students will not become overwhelmed by drawing all of the existing figures in each pattern. Circulate as teams work on this problem and encourage teams to discuss what they are noticing in each pattern. Facilitate discussions by asking, “How is the pattern growing?” “How can you show that growth in the table? In the graph?” and “What would figure 100 look like?”

**Team Strategies:**

This might be a time to do a Participation Quiz to help teams work together more effectively.

During Lessons 4.1.2 and 4.1.3, students are instructed to use different colors to represent growth for the different tile patterns. To make sure that each team understands what to do, you may want to call all of the Facilitators to one location of the classroom for a Huddle. Explain how they should use a different color for each tile pattern and that this will help them find connections between the tile pattern, the rule, and the graph in Lesson 4.1.3. When these students return to their teams, it is their responsibility to make sure their team members understand how to use the different colors.

**Homework:**

Problems 4-17 through 4-21

**Notes to Self:**
4.1.2 How does it grow?

Seeing Growth in Different Representations

In Lesson 4.1.1, you looked at four different ways of representing patterns and began to find connections between them.

Throughout this chapter, you will explore connections between and find efficient ways to move from one type of representation to another. Today, you will look for specific connections between geometric patterns and equations.

As you work, keep these questions in mind:

How can you see growth in the rule?
How do you know your rule is correct?
What does the representation tell you?
What are the connections between the representations?

At the end of this lesson, put your work from today in a safe place. You will need to use it during Lesson 4.1.3.

4-12. Tile Pattern #1:

Examine the tile pattern shown at right and on the Lesson 4.1.2 Resource Page that your teacher gives you.

a. What do you notice? After everyone has had a moment to examine the figures independently, discuss what you see with your team.

b. Sketch the next figure in the sequence (Figure 4) on your resource page. Sketch Figure 0, which is the figure that comes before Figure 1. [See the “Suggested Lesson Activity” for Figures 0 and 4. Figure 4 has 18 tiles. Figure 0 has 2 tiles.]

c. How is the tile pattern growing? Where are the tiles being added with each new figure? On your resource page, use a marker or colored pencil to color in the new tiles in each figure. [The pattern is growing by 4 tiles, see Suggested Learning Activity for example shading of pattern.]

d. What would Figure 100 look like? Describe it in words. How many tiles would be in the 100th figure? Find as many ways as you can to justify your conclusion. Be prepared to report back to the class with your team’s findings and methods. [Figure 100 has 402 tiles, the center block is 2 by 101 tiles and there is a diagonal set of tiles 100 tiles long extending from the top right and bottom left corners.]
4-13. For each of the patterns below, answer questions (a) through (d) from problem 4-12. Use color to shade in the new tiles on each pattern on your resource page. Choose one color for the new tiles in part (a) and a different color for the new tiles in part (b). [Tile Pattern #2: b: See Figures below, Figure 4 has 17 tiles, Figure 0 has 1 tile; c: The pattern grows by 4 tiles; d: Figure 100 has 401 tiles, a center block with 100 tiles extending out to the top, bottom, right and left. Tile Pattern #3: b: See Figures below, Figure 4 has 19 tiles, Figure 0 has 3 tiles; c: the pattern grows by 4 tiles; d: Figure 100 has 403 tiles, two center tiles with a stack of 101 tiles on the left and 100 columns of 3 tiles on the right.]

a. Tile Pattern #2:

b. Tile Pattern #3:

4-14. PUTTING IT TOGETHER

Look back at the three different tile patterns in problems 4-12 and 4-13 to answer the following questions.

a. When you compare these three patterns, what is the same and what is different? Explain in a few sentences. [They each have the same growth factor, but they start with different numbers of tiles.]

b. Find an equation (rule) for the number of tiles in each pattern. Label each tile pattern on your resource page with its rule. [#1: \( y = 4x + 2 \), #2: \( y = 4x + 1 \), #3: \( y = 4x + 3 \)]

c. What connections do you see between your equations and the tile pattern? Show and explain these connections. [Students should notice that a growth of 4 tiles corresponds to the \( 4x \) in each equation and that the starting number of tiles is added on to the rule.]

d. Imagine that the team next to you created a new tile pattern that grows in the same way as the ones you have just worked with, but they refused to show it to you. What other information would you need to be able to predict the number of tiles in Figure 100? Explain your reasoning. [You would need to know the number of tiles in Figure 0.]

Chapter 4: Multiple Representations
4-15. Consider **Tile Pattern #4**, shown below.

a. Draw Figures 0 and 4 on the resource page. [**Figure 0 has 3 tiles, Figure 4 has 11 tiles.**]

b. Find an equation (rule) for the number of tiles in this pattern. On your resource page, label Tile Pattern #4 with its rule. Then use a new color to show where the numbers in your rule appear in the tile pattern. [\( y = 2x + 3 \)]

c. What is the same about this pattern and Tile Pattern #3? What is different? What do those similarities and differences look like in the tile pattern? In the equation? [**They both have 3 tiles in Figure 0, but they grow by different amounts.**]

d. The **growth factor** is the number of tiles by which the pattern increases each time you move from one figure to the next figure in the sequence. How is the growth factor represented in each equation? [**The growth factor is the coefficient of the \( x \)-term.**]

*Remember to put your work from today in a safe place, because you will need to use it during the next lesson.*

4-16. **LEARNING LOG**

For today’s Learning Log entry, draw a web of the different representations, starting with the diagram below. Draw lines and/or arrows to show which representations you have studied so far in this course. Explain the connections you learned today. Be sure to include anything you figured out about how the numbers in equations (rules) relate to tile patterns. Title this entry “Representations of Patterns Web” and label it with today’s date.
4-17. Simplify each of the following equations and solve for \( x \). Show all work and check your solution.

\[ a. \quad 7 - 3x = -x + 1 \quad [ x = 3 ] \]
\[ b. \quad -2 + 3x = -(x + 6) \quad [ x = -1 ] \]

4-18. Leala can write a 500-word essay in an hour. If she writes an essay in 10 minutes, approximately how many words do you think the essay contains? [ about 83 words ]

4-19. Copy and complete the table below.

<table>
<thead>
<tr>
<th>IN ((x))</th>
<th>2</th>
<th>10</th>
<th>6</th>
<th>7</th>
<th>–3</th>
<th>5</th>
<th>–10</th>
<th>1000</th>
<th>( x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>OUT ((y))</td>
<td>9</td>
<td>25</td>
<td>17</td>
<td>19</td>
<td>–1</td>
<td>15</td>
<td>–15</td>
<td>2005</td>
<td>( 2x + 5 )</td>
</tr>
</tbody>
</table>

\[ a. \quad \text{Explain in words what is done to the input value (} x \text{) to produce the output value (} y \text{). } [ \text{Multiply } x \text{ by 2 and add 5.} ] \]
\[ b. \quad \text{Write the rule you described in part (a) with algebraic symbols. } [ y = 2x + 5 ] \]

4-20. When Susan’s brother went to college, she and her two sisters evenly divided his belongings. Among his possessions were 3 posters, 216 books, and 24 CDs. How were these items divided? [ Each sister received 1 poster, 72 books, and 8 CDs. ]

4-21. Kelso’s mom wants to put a floating blanket over the family’s circular wading pool to keep the heat in and the leaves out. The pool has a diameter of 10 feet.

\[ a. \quad \text{How many square feet of blanket will Kelso’s mother need? } [25\pi = 78.5 \text{ sq ft}] \]
\[ b. \quad \text{If the pool supply store charges } \$0.10 \text{ per square foot for the blanket, how much will the material for the blanket cost? } [ \$7.85] \]
Lesson 4.1.3  How does it grow?

Connecting Linear Rules and Graphs

Lesson Objective: Students will connect linear geometric patterns with patterns on a graph, specifically focusing on how a geometric pattern grows and how the size of Figure 0 can be determined from information on a graph.

Mathematical Practices: Today students look for and express regularity in repeated reasoning as they identifying how the growth pattern and Figure 0 relate to a linear equation. By writing an equation for a pattern, they are modeling with mathematics.

Length of Activity: One day (approximately 45 minutes)

Core Problems: Problems 4-22 and 4-23

Materials: Markers or colored pencils
Lesson 4.1.3 Resource Page, one copy per student and one for board display

Suggested Lesson Activity: It may be helpful, especially if you did not complete the entry together as a class, to begin class by having one or two students read aloud from their Learning Log entries from the previous lesson, sharing the connections they noticed or the questions they have yet to answer.

Also, if your students did not have enough time to start (or finish) Tile Pattern #4 from problem 4-15 in Lesson 4.1.2, have them start with that problem today.

Once students are ready to move on, ask a student to read the introduction before problem 4-22 and distribute the Lesson 4.1.3 Resource Page. In problem 4-22, students will use data from the tile patterns they investigated in Lesson 4.1.2 to create $x \rightarrow y$ tables and graphs of those patterns. Before they begin working, remind students to continue the color-coding system they began in Lesson 4.1.2 as they make their $x \rightarrow y$ tables and graphs. This will facilitate their seeing connections between the different representations. (If colored pencils or markers are unavailable, remind students to label their representations clearly.)

When students have finished problem 4-22, lead a whole-class discussion about what teams concluded. It is recommended that as students state their teams’ conclusions, you list the conclusions on the board. This will allow you to come back to these conclusions easily during the rest of the unit. You could also have the students share using a Traveling Salesman strategy before you have a whole class discussion.

Start by focusing on the idea of growth, and familiarize students with the term “growth factor.” If students did not draw and label a “growth
triangle” on the graph (see example at right), this is a good time to share this technique with them. Note that this is a preview of future work with slope. At this time, you want students to see slope as a growth factor that can be seen in a rule, pattern, table, or graph. Students will work more formally with slope in Chapter 7.

Also ask students about the “other number” in the rule. Encourage students to come up and show where they see this number in each representation. Students will continue to investigate $b$ in Lesson 4.1.4, so they do not need to have complete answers at this time.

When the discussion is complete, ask teams to work on problem 4-23 for about 10 minutes. Then bring the class back together to have students share their answers and justifications in a class discussion.

**Closure:**
(10 minutes)

Use problem 4-24 to prompt students to answer the target questions in their Learning Logs. Allow plenty of time, as this consolidation is extremely important. It is suggested that you consider using a Walk and Talk strategy before the students begin to write.

**Universal Access:**

**Academic Literacy and Language Support:** Have students draw growth triangles as explained in the Suggested Lesson Activity for problem 4-22. It is important for students to connect the growth triangle to its representation in the pattern. Incorporate into the class discussion that the growth triangle represents that each consecutive figure in the pattern grows by 4 tiles.

**Team Strategies:** If team interactions are lagging, consider beginning a lesson by discussing the kinds of comments or questions that could be useful during team conversations for the particular activities of the day. If students understand the nature of the coming task, you could invite the class to brainstorm ideas for sentence starters and then share them using a Whiparound. Possible ideas are:

- *What if we tried ______________?*
- *I have another approach to the problem. How about ______________?*
- *I’m not sure that will work because ______________. What if we tried ______________?*
- *Can anyone suggest a different approach?*
- *Your idea makes me think about ______________.*
- *Could you explain that another way?*
- *I hear you saying ______________. Is that right?*
- *I like your idea that ______________.*
You could then post the list of student-generated sentence starters in the classroom so that students can refer to them as they work together.

When you observe high quality interactions, commend the team members and share your observations with the class.

**Homework:** Problems 4-25 through 4-29

**Notes to Self:**
4.1.3 How does it grow?

Connecting Linear Rules and Graphs

You have been looking at geometric patterns and ways in which those patterns can be represented with $x \rightarrow y$ tables, graphs, and equations. In Lesson 4.1.2, you worked with four different tile patterns and looked for connections between the geometric shapes and the numbers in the equations. Today you will go back to those four equations and look for connections to other representations.

By the end of this lesson, you should be able to answer these target questions:

- How is growth shown in a graph?
- How is growth shown in a rule?
- How can you determine the number of tiles in Figure 0 from a graph?
- How can you determine which tile pattern grows faster from a graph?


a. Make sure you have a rule for each tile pattern.

\[
\begin{align*}
\text{Pattern #1:} & \quad y = 4x + 2, \\
\text{Pattern #2:} & \quad y = 4x + 1, \\
\text{Pattern #3:} & \quad y = 4x + 3, \\
\text{Pattern #4:} & \quad y = 2x + 3
\end{align*}
\]

b. Complete the table for each rule. [See bold answers in tables below.]

<table>
<thead>
<tr>
<th>Tile Pattern #1</th>
<th>Tile Pattern #2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Figure #</strong></td>
<td>0 1 2 3 4</td>
</tr>
<tr>
<td><strong># of Tiles</strong></td>
<td>2 6 10 14 18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tile</th>
<th>Figure #</th>
<th>0 1 2 3 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong># of Tiles</strong></td>
<td>1 5 9 13 17</td>
</tr>
<tr>
<td></td>
<td><strong># of Tiles</strong></td>
<td>3 5 7 9 11</td>
</tr>
</tbody>
</table>

Problem continues on next page. →

Chapter 4: Multiple Representations
4-22.  Problem continued from previous page.

d. Explain how the growth appears in the pattern, in the table, in the graph, and in the rule.  [In the pattern, the growth is represented by the added tiles.  In the rule, the growth shows up as the coefficient of \( x \). In the table, the number of tiles goes up 4 for each time the pattern number increases by one. In the graph, the growth can be shown as the vertical change between the points for each figure.]

e. What connections do you see between these representations? Describe any connections you see.  [Some possible responses include: “If the graph goes in the same direction, they all add the same amount”; “the pattern in the table (growth) is the number multiplied by \( x \) in the rule”; “the first number for Figure 0 is always the last number in the rule”; “if we have the rule, we have a shortcut to graph by starting with Figure 0 (end #) and going up by the growth number.”]

4-23. The graph at right gives information about three new tile patterns. Remember that in this course, tile patterns will be considered to be elements of continuous relationships and thus will be graphed with a continuous line or curve.

Answer the following questions as a team.

a. What information does the circled point (○) on the graph tell you about tile pattern A? [Figure 4 has 10 tiles.]

b. Find the growth of each tile pattern. For example, how much does tile pattern A increase from one figure to the next? Explain how you know. [A and B both increase by 2, while C increases by 4.]

c. Look at the lines for tile patterns A and B. What is the same about the two lines? What conclusion can you make about these tile patterns? What is different about the lines? What does this tell you about the tile patterns? Use what you see on the graph to justify your answers. [The lines are parallel. Tile patterns A and B must grow by the same amount for each figure. The lines start at different values on the y-axis. Figure 0 for tile patterns A and B must have a different number of tiles (2 and 8, respectively).]

d. Look at lines A and C on the graph. What do these two lines have in common? In what ways are the lines different? What does this tell you about the tile patterns? Explain completely. [Lines A and C have a different “steepness.” They start at the same value on the y-axis. Tile patterns A and C grow by different amounts for each figure; Figure 0 for both patterns must have the same number of tiles.]
4-24. LEARNING LOG

In your Learning Log, answer the target questions for this lesson, reprinted below.

*How is growth shown in a graph?*

*How is growth shown in a rule?*

*How can you determine the number of tiles in Figure 0 from a graph?*

*How can you determine which tile pattern grows faster from a graph?*

Be sure to include at least one example. Title this entry “Connecting Linear Rules and Graphs” and label it with today’s date.

4-25. Two of the connections in your Representations of Patterns Web are pattern → table and pattern → rule. Practice these connections as you answer the questions below.

a. On graph paper, draw Figure 0 and Figure 4 for the pattern at right. [See figures at right below.]

b. Represent the number of tiles in each figure with a table. [See table at right.]

c. Represent the number of tiles in each figure with an algebraic rule. [y = 2x + 3]

<table>
<thead>
<tr>
<th>Figure #</th>
<th># of Tiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
</tr>
</tbody>
</table>

4-26. Use the formula for the area of a circle to solve for the radius of the circle if the area is 78.5 cm². [5 cm]

4-27. For each of the equations below, solve for x. Show all work and check your solution.

a. −2 + 2x = −x + 2 + x  [x = 2]  
b. 2 − 3x = x + 2  [x = 0]
4-28. Another one of the connections in your Representations of Patterns Web is graph → table. In Chapters 1 through 3, you developed tools to find a table from a graph. Consider this connection as you complete the table below. The table is based on the graph at right.

<table>
<thead>
<tr>
<th>IN (x)</th>
<th>–3</th>
<th>–2</th>
<th>–1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>OUT (y)</td>
<td>–2.5</td>
<td>–2</td>
<td>–1.5</td>
<td>–1</td>
<td>–0.5</td>
<td>0</td>
<td>0.5</td>
</tr>
</tbody>
</table>

4-29. Joe drove 100 miles from San Francisco to Gilroy and used 4 gallons of gas. How much gas should he expect to use for a 3000-mile trip to New York City? What is the unit rate (miles per gallon)? Be sure to justify your reasoning.

\[
\frac{100}{4} = 25 \text{ (unit rate)}, \quad \frac{3000}{25} = 120 \text{ gallons}
\]
**Lesson 4.1.4  What is the rule? How can I use it?**

**y = mx + b**

**Lesson Objective:** Students will develop new connections between multiple representations of patterns and identify rules for these patterns using the $y = mx + b$ form of a linear equation. Students will then apply their understanding of $m$ as growth factor and $b$ as Figure 0 or the starting point of the pattern to write a rule from a graph and to create a pattern based on a linear rule.

**Mathematical Practices:** Students will **make use of the structure** of a linear equation to write a rule from a graph.

**Length of Activity:** One day (approximately 45 minutes)

**Core Problems:** Problems 4-40 through 4-34

**Materials:** None

**Suggested Lesson Activity:** Begin this lesson with a class discussion to summarize where the class is in its investigation of multiple representations of patterns. Use the class poster of the Representations of Patterns Web from Lesson 4.1.2 to show which connections are strong at this point. Once updated, the web should look this:

Note that the arrow from table to rule is still dashed. This indicates that this connection is still being developed. Today students will complete this connection in problem 4-33 as they examine how to write rules from tables using growth and the number of tiles in Figure 0.

Referring to the work from Lessons 4.1.2 and 4.1.3, ask students to list some of the rules with which they have been working. Ask, “**What do these rules have in common?**” Students may notice that each rule has a constant term added, or that they each have $x$ times another number, etc.

While problems 4-30 and 4-31 are written so they can be used in teams, it is **highly recommended** that you hold a whole-class discussion based on this problem at the beginning of class. Structuring this as a Think-Pair-Share will allow all students to enter their team and whole class discussion with something they have already thought about, and give them something to talk about. Introduce the $y = mx + b$ equation and write it on the board. Draw students’ attention to the fact that all of the equations (rules) they have been writing are in this form. Tell them that $x$ and $y$ still represent variables, but that $m$ and $b$ represent parameters (numbers that stay the same in the equation after they are
chosen). Ask, “If this was the equation of the tile pattern, what information would you get from each part of this equation? For example, what does m tell you about the equation?” Draw arrows to m and b and label what each stands for. If students are not sure at this point, refer them back to their Learning Log entries and classwork from Lessons 4.1.2 and 4.1.3. This is the first opportunity for students to check the conclusions that they have been recording in their Learning Logs, so take time to ask students to share examples of what they mean by “growth factor” and “starting value” or “Figure 0.”

Explain that in problems 4-32, 4-33, and 4-34, students will use their ideas about m and b and what they know about the different representations of patterns to try to find more connections on the web. Specifically, this lesson will develop the connections graph → rule and rule → pattern, and will further develop the connection table → rule. You could do these problems using a Red Light, Green Light strategy. Or you could do a Hot Potato for problem 4-33. Either strategy will encourage the teams to keep on task.

If time allows, problem 4-35 asks students to invent two tile patterns with the same growth factor but different total numbers of tiles to compare the rules for each pattern. This requires them to consider the impact of the number of tiles in Figure 0 and of b.

**Closure:**
(5 minutes) After students complete this lesson’s problems, problem 4-36 prompts them to record their understanding of m and b in their Learning Logs. This allows students to synthesize in writing the ideas that m is a growth factor and b is a starting value or the area of Figure 0.

**Team Strategies:** A Fortune Cookie can be done at anytime throughout a chapter so that students can share ideas. Develop a list of five or six sentence starters to use as “fortunes” for the activity. The topics should focus on chapter topics, general ideas or team norms that need to be discussed. The sentence starters need to be copied, cut apart and one set of each put into envelopes. Each table gets an envelope. One idea is to have the students share study skills such as:

- When studying for a test, I…
- When doing homework, I…
- To keep my notebook (Learning Log, Journal) organized, I…
- When working with my teammates, I…
- When using my Team Role, I (be specific about your role)…

**Homework:** Problems 4-37 through 4-41

**Notes to Self:**
4.1.4 What is the rule? How can I use it?

\[ y = mx + b \]

In Lessons 4.1.2 and 4.1.3, you investigated connections between tile patterns, \( x \rightarrow y \) tables, graphs, and rules (equations). Today you will use your observations about growth and Figure 0 to write rules for linear patterns and to create new tile patterns for given rules.

4-30. With your team, list some of the equations you have been working with in the past two lessons. What do all of these rules have in common? [They all have something added to and/or multiplied by \( x \).]

4-31. UNDERSTANDING \( y = mx + b \)

Rules for linear patterns can all be written in the form \( y = mx + b \). In \( y = mx + b \), \( x \) and \( y \) represent variables, while \( m \) and \( b \) represent parameters. Parameters are numbers that stay the same in the equation after they are chosen. Discuss these questions with your team:

a. What does \( m \) tell you about the pattern? [How each figure changes.]

b. What does \( b \) tell you about the pattern? [The number of tiles in Figure 0.]

4-32. GRAPH \( \rightarrow \) RULE

Allysha claims she can find the equation of a line by its graph without using a table. How is that possible? Discuss this idea with your team and then try to find the equation of the line at right without first making a table. Be ready to share with the class how you found the rule. [\( y = 3x + 1 \)]
4-33. **TABLE → RULE**

Allysha wonders if she can use the idea of \( m \) and \( b \) to find the equation of a line from its table.

a. For example, if she knows the information about a linear pattern given in the table below, how can she find the equation of the line? Work with your team to complete the table and find the rule. \([ \ y = 5x - 2 \ ]\)

<table>
<thead>
<tr>
<th>IN ((x))</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>OUT ((y))</td>
<td>–2</td>
<td>3</td>
<td>8</td>
<td>13</td>
<td>18</td>
<td>23</td>
<td>28</td>
</tr>
</tbody>
</table>

b. Use this same idea to find the rule of the linear tile patterns represented by the tables below. \([ \text{a: } y = 2x + 5, \text{ b: } y = -3x + 7 \ ]\)

i.  

<table>
<thead>
<tr>
<th>IN ((x))</th>
<th>–1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>OUT ((y))</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>11</td>
<td>13</td>
<td>15</td>
</tr>
</tbody>
</table>

ii.  

<table>
<thead>
<tr>
<th>IN ((x))</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>OUT ((y))</td>
<td>7</td>
<td>4</td>
<td>1</td>
<td>–2</td>
<td>–5</td>
<td>–8</td>
<td>–11</td>
</tr>
</tbody>
</table>

c. Write a summary statement explaining how you used your knowledge about \( m \) and \( b \) to quickly write a rule. \([ \text{Sample response: } m \text{ is the amount that the } y \text{ value changes when } x \text{ changes by } 1 \text{ in the table, } b \text{ is the } y-\text{value when } x \text{ is } 0. \]}

4-34. **RULE → PATTERN**

In each problem below, invent your own pattern that meets the stated conditions. \([ \text{Answers vary.} \]}

a. Draw Figures 0, 1, 2, and 3 for a tile pattern that has \( y = 4x + 3 \) as a rule.

b. A tile pattern decreases by 2 tiles. Figure 2 of the pattern has 8 tiles. Draw Figures 0, 1, 2, and 3 and write the rule (equation) for the pattern.

4-35. Invent two different tile patterns that grow by 4 every time but have different \( x \rightarrow y \) tables. Draw Figures 0, 1, 2, and 3 and find rules for each of your patterns. What is different about your rules? What is the same? \([ \text{Patterns vary, but should have rules with } 4x \text{ and different } y-\text{intercepts } (b). \]
LEARNING LOG

The linear equations you have been working with can be written in this general form: \( y = mx + b \)

In your Learning Log, summarize what you know about \( m \) and \( b \) so far. What does the \( m \) tell you about a pattern? What does the \( b \) tell you about a pattern? Where can you see \( m \) and \( b \) in each representation? Sketch examples if it helps. Title this entry “\( y = mx + b \)” and label it with today’s date.

Examine the \( x \rightarrow y \) table at right.

a. Invent a tile pattern that fits this data.  
   [Answers vary.]

b. What is the pattern’s growth factor? Show where the growth factor appears in the \( x \rightarrow y \) table and the tile pattern.  
   [Each figure has 4 more tiles than the figure before it.]

c. Write a rule for this pattern.  
   [\( y = 4x + 5 \)]

Look at the graph at left. What statements can you make about the tile pattern the graph represents? How many tiles are in Figure 0? Figure 1? What is the growth factor? What is the rule for the pattern? 
[Figure 0 has 7 tiles. Figure 1 has 9 tiles. The growth factor is 2. The rule is \( y = 2x + 7 \).]
4-39. For each equation below, solve for $x$. Check your solution, if possible, and show all work.

a. $3x - 6 + 1 = -2x - 5 + 5x$
   [All numbers]

b. $-2x - 5 = 2 - 4x - (x - 1)$
   [$x = \frac{3}{2}$]

4-40. I am thinking of a number. When I double my number and then subtract the result from five, I get negative one. What is my number? Write and solve an equation.
   [If $x$ is my number, then $5 - 2x = -1$; $x = 3$.]

4-41. On your paper, copy the table below and use your pattern skills to complete it.

<table>
<thead>
<tr>
<th>IN ($x$)</th>
<th>2</th>
<th>10</th>
<th>5</th>
<th>-5</th>
<th>4</th>
<th>-3</th>
<th>1.5</th>
<th>50</th>
<th>$x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OUT ($y$)</td>
<td>4</td>
<td>28</td>
<td>13</td>
<td>-17</td>
<td>10</td>
<td>-11</td>
<td>2.5</td>
<td>148</td>
<td>$3x - 2$</td>
</tr>
</tbody>
</table>

a. Explain in words what is done to the input value, $x$, to produce the output value, $y$. [Multiply $x$ by 3 and subtract 2.]

b. Explain the process you used to find the missing input values.
Lesson 4.1.5 What are the connections?

Checking the Connections

Lesson Objective: Students will apply their understanding of growth, Figure 0, and connections between multiple representations to situations where they are presented with disparate pieces of information and must generate a complete pattern. Students will apply their understanding of growth and Figure 0 to new contexts in order to generate complete representations.

Mathematical Practices: In this two-day lesson students use the appropriate tool (Representations of a Patterns Web) strategically to make sense of problems and persevere in solving them.

Length of Activity: Two days (approximately 90 minutes)

Core Problems: Parts (a) and (b) of problem 4-42

Materials: General Team Roles Resource Page (optional)

Suggested Lesson Activity: Begin the day by reviewing how far the class has gotten in completing the Representations of Patterns Web using the class web begun in Lesson 4.1.2.

Quickly orient students to the activity described in problem 4-42. Students will be working with their teams to make sense of information from different parts of the web in order to generate a complete pattern. Highlight that these are challenging problems that require team members to talk together, share ideas, question each other, and justify their statements. Remind students that they may look back at their Learning Log entries, class assignments, and the focus questions for the chapter to help them. You can use a Teammates Consult to initiate the conversation.

You are strongly encouraged to start teams quickly on challenge problem 4-42. Although there are only four problems in this lesson, students will need time to make sense of the pattern and generate all four complete representations. The four problems may be completed in any order. Since the graph in part (b) is revisited in Chapter 7 as an introduction to slope, you may want to instruct all teams to analyze this tile pattern. It is recommended that you create a system for checking that requires teams to explain one problem fully before moving on to the next problem.

As teams work, you will need to circulate and offer encouragement. If teams are stuck, ask questions like, “What information can you read in this representation?”, “What information do you need in order to figure out the pattern? Where would you look for that information?”, “How are these two representations related?”, “How can you check that your representations match?” and “How do you see the growth?” Make note of teams with particularly clear explanations, if you want a team to present its work at the end of the second 50-minute “day.”
Allow plenty of time for closure (see below). You can allow students to use an I Spy strategy if their team is stuck. You can also call a Huddle at any time if you want to share a hint with all teams.

**Closure:**  
*(20 minutes)*

The team challenge, problem 4-42, is a two-day activity. Be aware of the time and bring the class together before the end of the first day’s lesson for a brief discussion of where teams are and which parts are most challenging to them. As the second day begins, remind students what the goals of the activity are and get them started right away.

Teams will take different amounts of time to answer these questions. As teams finish, select a problem for each team to present to the class. It is not necessary for every team to present; even having two problems presented will create opportunities for teams to share strategies and ways of seeing the connections. (If timing will not allow for presentations, ask teams to create stand-alone posters of their solutions and do a Gallery Walk.) It is also not necessary for every team to finish all four parts, (a) through (d), of problem 4-42. Teams should have created complete multiple representations of at least two of the problems and should have made substantial progress on a third problem before listening to presentations or moving on to the next lesson. You could also use a Swapmeet to share solutions.

Save 5 to 10 minutes at the end of class for students to reconsider the Representations of Patterns Web. Problem 4-43 asks students to update the web they created in problem 4-16, drawing in solid arrows only when they are confident that they can move directly between those representations.

After students have updated their own webs, you may want to have them talk in their teams or as a class to remind each other of connections they may have forgotten. At this point, a student’s web could look like the one at right.

Then pull the class together and update the class Representations of Patterns Web if necessary.

**Universal Access:**  
**Academic Literacy and Language Support:** Problem 4-42 shows partial representations of different patterns. To support ELL and students needing additional support, provide the following sentence starters on the board. Have teams use the sentence starters to facilitate their discussions on each pattern. “The representations shown for this pattern are...” “From the representations shown I know figure number ___ has ___ tiles. Based on this information I can conclude the growth for this pattern is...” “I determined the growth for the pattern by...”
In “Review and Preview” problem 4-49, students will be translating a story into a graphical representation. ELL students may need additional support in expressing the vocabulary in graphical form. If necessary, have a copy of the graph on the board with the axes labeled as shown in the problem. Direct a class discussion with the following questions. “If a car is going along a road at the same speed for several blocks, how can we represent this with a line?” “What if the car is driving at the same speed for several miles until it runs into traffic and has to slow down?” What if the car is driving at the same speed and then picks up speed in order to pass several cars. What kind of line would represent the car’s travel?”

Team Strategies: If your teams are “stuck in rut” in their conversations, you could provide sentence starters, that all the students can use, regardless of role, like the ones below.

- I noticed that …
- Why did you …?
- This reminds me of …
- Did any one else get …?

Homework: Day 1: Problems 4-44 through 4-48
Day 2: Problems 4-49 through 4-53

Notes to Self:
4.1.5 What are the connections?

Checking the Connections

In the last several lessons, you have been finding connections and relationships between different representations of patterns. You have worked backward and forward. You have also used information about Figure 0 (or the starting point) and the growth factor to write rules. In today’s activity, you will use pieces of information from various parts of the web to generate a complete pattern.

4-42. CHECKING THE CONNECTIONS: TEAM CHALLENGE

Today you are going to apply what you know about the starting point (Figure 0), the growth factor, and the connections between representations to answer some challenging questions. The information in each question, parts (a) through (d), describes a different pattern. The graph of each pattern is a line. From this information, generate the rule, \(x \rightarrow y\) table, graph, and tile pattern (Figures 0 through 3) that follow the pattern. You may answer these questions in any order, but make sure you answer each one completely before starting another problem.

Work together as a team. The more you listen to how other people see the connections and the more you share your own ideas, the more you will know at the end of the lesson. Stick together and be sure to talk through every idea.

Each person will turn in his or her own paper at the end of this activity, showing four complete representations for each pattern. Your work does not need to be identical to your teammates’ work, but you should have talked and agreed that all explanations are correct. [Rules for patterns: a: \(y = 3x + 4\), b: \(y = 9x - 5\), c: \(y = -2x + 18\), d: \(-3x + 7\) (given).]
4-42.  

Problem continued from previous page.

a.  

![Figure 3](image)

b.  

![Graph](image)

c.  

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Tiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

![Figure 8](image)

d.  

\[ y = -3x + 7 \]
4-43. REPRESENTATIONS OF PATTERNS WEB

Update your Representations of Pattern Web from problem 4-16 with any new connections that you can make between representations following today’s work. Pay attention to the direction of any arrows that you draw.

4-44. Complete a table for the rule \( y = 3x - 2 \). [ See a possible solution table below. ]

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
x & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 \\
\hline
y & -14 & -11 & -8 & -5 & -2 & 1 & 4 & 7 & 10 \\
\hline
\end{array}
\]

a. Draw a complete graph for this rule.

b. Is \((-50, -152)\) a point on the graph? Explain how you know.
   [ Yes. \(-152 = 3(-50) - 2\) ]

4-45. Write down everything you know about the tile pattern represented by the \(x\rightarrow y\) table at right. Be as specific as possible. [ Answers vary. “Figure 4 has 32 tiles,” “each figure has 7 more tiles than the previous figure,” “Figure 0 has 4 tiles,” and “the rule is \(y = 7x + 4\).” ]

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
3 & 25 \\
5 & 39 \\
6 & 46 \\
1 & 11 \\
\hline
\end{array}
\]

4-46. Find the area and circumference of a circle that has a diameter of 17 mm. Write your answers in terms of \(\pi\) and as a decimal approximation.
   [ \( A = 72.25\pi \approx 226.98 \text{ mm}^2 \), \( C = 17\pi \approx 53.4 \text{ mm} \) ]

4-47. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right. [ Find solutions in the diamonds below. ]

a.  
\[
\begin{array}{c}
\frac{1}{2} \\
2.5 \\
1 \\
2 \\
\end{array}
\]

b.  
\[
\begin{array}{c}
10 \\
-2 \\
-5 \\
-7 \\
\end{array}
\]

c.  
\[
\begin{array}{c}
5 \\
-5 \\
-1 \\
-6 \\
\end{array}
\]

d.  
\[
\begin{array}{c}
-112 \\
8 \\
-14 \\
-6 \\
\end{array}
\]
4-48. Simplify each of the expressions below. You may use an Equation Mat and tiles.

a. \(- (5x + 1) \ [ -5x - 1 ]\)  
b. \(6x - (-5x + 1) \ [ 11x - 1 ]\)  
c. \(-(1 - 5x) \ [ 5x - 1 ]\)  
d. \(-5x + (x - 1) \ [ -4x - 1 ]\)

4-49. Invent a tile pattern that grows by 4 each time. Draw Figures 0, 1, 2, and 3. Use color or shading to show the growth. [Answers vary.]

4-50. Complete a table for the rule \(y = 3 - x\).

a. Draw a complete graph for this rule. [The line should pass through (3, 0) and (0, 3).]  
b. Is (32, -29) a point on this graph? Explain why or why not. [Yes. \(-29 = 3 - 32\)]

4-51. For each equation below, solve for the given variable. Check your solutions, if possible, and show all work.

a. \(3p - 7 + 9 - 2p = p + 2\), solve for \(p\)  
   [Any number]  
b. \(-2x + 5 + (-x) - 5 = 0\), solve for \(x\)  
   [\(x = 0\)]  
c. \(12 = r + 6 - 2r\), solve for \(r\)  
   [\(r = -6\)]  
d. \(-(y^2 - 2) = y^2 - 5 - 2y^2\), solve for \(y\)  
   [No solution]

4-52. Solve each equation below for \(x\). Then check your solutions.

a. \(\frac{x}{8} = \frac{3}{4}\)  
   [\(x = 6\)]  
b. \(\frac{x}{5} = \frac{4}{40}\)  
   [\(x = 16\)]  
c. \(\frac{1}{8} = \frac{x}{12}\)  
   [\(x = 1.5\)]  
d. \(\frac{x}{10} = \frac{12}{15}\)  
   [\(x = 8\)]
4-53. Sketch a graph to match each story below using axes labeled as shown at right.

a. Luis rides his skateboard at the same speed all the way home. It takes him ten minutes to get there.
   [ This should be a horizontal line with a positive y-value that ends at \( x = 10 \). ]

b. Corinna jogs along at the same speed until she reaches a hill, and then she slows down until she finally stops to rest. [ This should start with positive y-value and have a slope of 0, then it should turn to negative slope, and then it should return to a slope of 0 at the x-axis. ]

c. Sergei is talking with his friends at the donut shop when he realizes that it is almost time for math class. He runs toward school, but he slows to a walk when he hears the bell ring and realizes that he is already late. He sits down in class four minutes after he left the donut shop.
   [ This should start horizontally along the line \( x = 0 \), then it should rise quickly to a higher horizontal line, and then it should descend to a lower positive horizontal line, ending at \( x = 4 \). ]
Lesson 4.1.6 How can I use growth?

Graphing a Line Without an $x \rightarrow y$ Table

**Lesson Objective:** Students will apply their knowledge of $m$ as growth factor and $b$ as Figure 0 or the starting value of a pattern to create graphs quickly without using an $x \rightarrow y$ table.

**Mathematical Practices:** In today’s lesson students **look for and make use of the structure** of a linear equation, using the **structure** to create a graph without making a table.

**Length of Activity:** One day (approximately 45 minutes)

**Core Problems:** Problems 4-54, 4-55, and 4-56

**Materials:** Graph paper for board display

**Suggested Lesson Activity:** Open class with the question, “**How can we use our knowledge of $y = mx + b$ to make graphs quickly?**” Explain that the goal of the lesson is to develop strategies for graphing patterns directly from a rule, without creating an $x \rightarrow y$ table. Some students may interpret this as meaning without actually **writing** an $x \rightarrow y$ table. If you see that students are not using the ideas of growth factor and Figure 0 to graph in problem 4-55, remind them to look for that information in the equation. They should be reversing the process used in problem 4-54.

Start teams on problems 4-54 through 4-56. Problem 4-54 prompts students to identify growth factor and the number of tiles in Figure 0 (starting point) from graphs in order to write rules (graph → rule). You could do this with a Pairs Check so that students can remind each other how to get a rule. You can also listen to the conversations of your students as an informal formative assessment as you circulate.

Problem 4-55 asks students to create graphs directly from rules (rule → graph). Before students begin this problem, they should have an emerging sense of using the $y$-intercept as the starting point of a graph and using a growth triangle to project the next point. Problem 4-55 gives students an opportunity to talk about this graphing strategy in teams to create a graph of each rule successfully.

Problem 4-56 asks students to create graphs and equations from information about patterns (pattern → graph, and either pattern → rule or graph → rule). As teams work, circulate and listen for talk about growth and Figure 0. Students should be making connections with the work they did in problems 4-54 and 4-55.

Once teams have completed problem 4-56, engage the whole class in a discussion of students’ strategies on the problems they have just completed. Have students come up to the board to “think aloud” through their graphing strategies, sharing their step-by-step decisions. At this point, discuss the term $y$-intercept as a class and elicit from students the
connections between \( b \) and the \( y \)-intercept. Bring out students’ ideas about what the growth factor and Figure 0 allow them to predict about a graph.

If time allows, problem 4-57 provides students with an opportunity to practice describing a pattern that fits a given rule.

Be sure to allow at least 15 minutes for closure.

**Closure:**
(15 minutes)

Part (a) of problem 4-58 asks students to update their Representations of Patterns Web. You may choose to do this together as a class. At this point, expect the web to look approximately like the one at right.

Ask, “What connections do we still need?” and let students know that they will be completing these connections in Lesson 4.1.7.

When students have finished creating the web, start them on part (b) of problem 4-58, which asks them to write instructions in their Learning Logs for graphing directly from a rule. The instructions should be written so that a younger student could follow them and create a graph *without creating an \( x \rightarrow y \) table*.

**Team Strategies:**
These activities require students to participate actively by sharing ideas and encouraging others to speak.

Remind students of the Collaborative Learning Expectations, in particular the “T” in TEAMS that represents “together, work to answer questions.”

The “E” represents “explain and give reasons.”

The “A” represents “ask questions and share ideas.”

The “M” represents “members of your team are your first resource.”

And the “S” represents “smarter together than apart.”

Remind students to be patient with each other, to welcome all ideas, and to invite each person in the team to speak so that they can have the benefit of each person’s ideas in their discussions.

**Homework:**
Problems 4-59 through 4-63

**Notes to Self:**
4.1.6 How can I use growth?

Graphing a Line Without an \(x\rightarrow y\) Table

You have now used your knowledge of growth factors and Figure 0 to create tile patterns and \(x\rightarrow y\) tables directly from rules. You have also looked at graphs to determine the equation or rule for the pattern. Today you will reverse that process and use an equation to create a graph without the intermediate step of creating an \(x\rightarrow y\) table.

4-54. For each of the graphs below:

- Write a rule.
- Describe how the pattern changes and how many tiles are in Figure 0.

a. \[ y = 2x + 2 \]

b. \[ y = 2x + 3 \]

c. \[ y = 8x + 5 \]

d. \[ y = -2x + 8 \]

e. \[ y = 4x + 8 \]

f. \[ y = -4x + 12 \]
4-55. Now reverse the process. Graph the following rules without first making a table. Parts (a) and (b) can go on the same set of axes, as can parts (c) and (d). Label each line with its equation, \( y \)-intercept (where it crosses the \( y \)-axis), and a growth triangle. 

\[ \text{See graphs below. a: (0, 3), b: (0, 0), c: (0, 8), d: (0, -1)} \]

\[
\begin{align*}
a. & \quad y = 4x + 3 \\
b. & \quad y = 3x \\
c. & \quad y = -3x + 8 \\
d. & \quad y = x - 1
\end{align*}
\]

4-56. Sketch a graph that fits each description below and then label each line with its equation. You can put all of the graphs on one set of axes if you label the lines clearly. Use what you know about growth factor and Figure 0 to help you.

\[
\begin{align*}
a. & \quad \text{A pattern that has three tiles in Figure 0 and adds four tiles in each new figure.} \\
& \quad [y = 4x + 3] \\
b. & \quad \text{A pattern that shrinks by three tiles between figures and starts with five tiles in Figure 0.} \\
& \quad [y = -3x + 5] \\
c. & \quad \text{A pattern that has two tiles in all figures.} \\
& \quad [y = 2]
\end{align*}
\]

4-57. Now reverse your process to describe the pattern represented by the rule \( y = -2x + 13 \). Be as detailed as you can. 

\[ \text{The pattern shrinks by two tiles between figures; Figure 0 has 13 tiles.} \]
4-58. CONSOLIDATING YOUR LEARNING

a. Find the Representations of Patterns Web that you updated at the end of Lesson 4.1.5. On it, add arrows for any new connections between representations that you can now make.

b. LEARNING LOG

In your Learning Log, write a step-by-step process for graphing directly from a rule. A student who has not taken this course should be able to read your process and understand how to create a graph. It may help you to think about these questions as you write:

*What information do you get from your rule?*

*How does that information show up on the graph?*

*Where does your graph start?*

*How do you figure out the next point?*

*What should you label to make it a complete graph?*

Title this entry “Graphing Without an $x \rightarrow y$ Table,” and label it with today’s date.

4-59. Use what you know about $m$ and $b$ to graph each rule below without making a table. Draw a growth triangle for each line. [See graphs below.]

a. $y = 2x - 3$

b. $y = -2x + 5$

c. $y = 3x$

d. $y = \frac{1}{2}x + 1$
4-60. Examine the graph below, which displays three tile patterns.

a. What do you know about Figure 0 for each of the three patterns? [ A and C have 0 tiles, and B has 30 tiles. ]

b. Which pattern changes most quickly? How quickly does it change? Show how you know. [ C grows most quickly at 10 tiles per figure. Its line is the steepest. ]

c. Which figure number has the same number of tiles in patterns B and C? Explain how you know. [ Figure 2. The lines intersect at (2, 20). ]

d. Write a rule for pattern B. [ \( y = -5x + 30 \) ]

4-61. Translate these algebraic statements into words: \( y = 2x + 5 \) and \( y = 6x + 5 \). [ The first can be stated, “y is twice \( x \) increased by 5,” and the second can be stated, “y is 6 times \( x \) increased by 5.” ]

a. What do you know about Figure 0 for each pattern? [ Both have 5 tiles. ]

b. Which pattern grows most quickly? How do you know? [ \( y = 6x + 5 \). \( m \) is greater. ]

4-62. Find the area and perimeter of each shape. Show your work. [ a: A = 72 sq cm, P = 48 cm, b: A = 37.7 sq. in., P = 26.85 in. ]

4-63. Ms. B is making snickerdoodle cookies. Her recipe uses one-and-a-half teaspoons of cinnamon to make two-dozen cookies. If she needs to make thirteen-dozen cookies to give one cookie to each of her students, how much cinnamon will she need? There are three teaspoons in one tablespoon. How many tablespoons and teaspoons will Ms. B need? [ \( 9 \frac{1}{2} \) teaspoons, which is 3 tablespoons and \( \frac{1}{4} \) teaspoon. ]
Lesson 4.1.7  What are the connections?

Completing the Web

Lesson Objective: Students will practice moving directly from one representation to another in the Representations of Patterns Web.

Mathematical Practices: This is the culminating lesson for using the Representations of Patterns Web. Students will make sense of problems involving moving between representations and persevere in solving these problems.

Length of Activity: One day (approximately 45 minutes)

Core Problems: Problems 4-64 and 4-65

Materials: Class set of graphing calculators or equivalent graphing technology

            Display graphing calculator or computer with display (optional)

            General Team Roles Resource Page (optional)

Technology Notes: Problem 4-65 is a great opportunity to reintroduce graphing technology. At this point, students should be very comfortable with drawing graphs and creating tables. Therefore, using the graphing calculators offers a chance for students to get reacquainted with a powerful graphing tool and to learn more about its features. If you do not have access to graphing technology, see the notes below.

Before class, be sure to set up and test your graphing technology to make sure batteries or software work as planned. For example, to ease the use of TI-83+/TI-84+ graphing calculators, be sure all are in “Function” mode and that all Stat Plots and functions are cleared. You may also want to make sure that the Table Set mode is set so that $\Delta x = 1$.

It also is a good idea to walk through problem 4-65 using the technology yourself on the day of the lesson to review the buttons and issues that may arise during the lesson.

If you do not have access to graphing technology: Students can still benefit by creating a table and graph by hand to determine which figure number has 79 tiles. However, without a graphing calculator, students will take a lot longer to scale axes and complete a table up to Figure 19.

Suggested Lesson Activity: Note: If you intend to have students use graphing technology for problem 4-65, do not distribute the technology until students finish problem 4-64. Introduce this lesson as an opportunity for students to solidify what they know and complete the Representations of Patterns Web. In Lesson 4.1.6, students added arrows to their webs from the previous lesson. Today they will test their knowledge and have an opportunity to work on
connections they are still unsure about.

Start problem 4-64 with Reciprocal Teaching. Have the first person talk about how the pattern in part (a) is changing/growing. Then have the second person do the same with part (b). The students are not to talk about what the figures might look like, but generally how many tiles are in each figure. They could also discuss the rule. Then after this brief discussion, they can pick up their pencils and begin to write about the problem.

Have teams spend 15 minutes on problem 4-64, which allows students to complete their web by creating the connections of graph → pattern and table → pattern.

Save at least 30 minutes for problem 4-65, which is a culminating activity for Section 4.1. Lead a brief discussion, pointing out that the Representations of Patterns Web is now complete. Encourage students to talk to teammates or to look for help outside of class for any connections they do not yet fully understand.

Start problem 4-65 by asking a student to read the directions for the problem aloud. This can be done with a Teammates Consult. Then distribute the graphing calculators or set students up on computers with graphing technology. Emphasize that each student is creating his or her own product that shows off what he or she has learned in this chapter. Outline your expectations for students’ final products; you may want to use class time for students to create rough drafts of their answers and have them complete final drafts for homework. Alternatively, you may want to encourage students to work slowly and carefully to create neat, clear products to be turned in at the end of class.

Expect questions about how to use the graphing technology. You may want to spend 5 minutes with a quick reminder of how to graph a rule, how to change the window, and how to view a table for a given rule. If you use team roles, you can also call up Resource Managers for a “huddle” to go over technology details so that they can go show their teams.

Circulate as students work. Remind students that they will want to show and explain how they created each representation. While the graphing calculator will graph the line for a given rule, ask questions that help you know if students understand the result, such as, “How do you know if that line is correct?” or “How would you graph the line without the graphing calculator?” Encourage students to plan ahead as they work on problem 4-65 so that they can clearly show in each representation which figure number will have 79 tiles. This will require them to pay careful attention to how they set their window for the graph.

Students may not immediately see how to use the rule (equation) for the pattern to show which figure will have 79 tiles. Ask questions to help them understand the relationship between the equation and the number of
tiles in a figure. Ask, “How can you use your rule to show which figure will have 79 tiles in it?” and “What part of the equation gives you information about the number of tiles in the figure?”

Problem 4-66 offers a challenge for teams that finish problem 4-65 early. It provides students the x-intercept and growth of a graph and asks them to find a rule. Some students may mistakenly use the x-intercept as they would the y-intercept. Remind them that they can test their rule on their graphing calculators and can see if the line indeed has an x-intercept at (2, 0).

**Closure:**
(5 minutes)
Pull the class together with a brief closing discussion about what they accomplished during Section 4.1. Ask students questions that require them to consolidate their understanding, such as, “Do you think these connections only work for linear patterns? Why or why not?” Let the students know that they will be using these new tools throughout the rest of the course. You could do this with a Walk and Talk instead.

**Universal Access:**  
**Extension:** Students may need support with problem 4-66. Guide students by asking questions such as, “If the pattern grows by 4 what do you know about the rule?” [ \( y = 4x + b \) ] “If the x-intercept is 2, what do you know about the y value at that point?” [ \( y = 0 \), so the point is \((0, 2)\) ] “Can you use these two pieces of information to help you find \( b \) in the rule \( y = mx + b \) ?”
\[
\begin{align*}
y & = 4x + b \\
0 & = 4(2) + b \\
0 & = 8 + b \\
b & = -8
\end{align*}
\]
For Review and Preview problem 4-67 part (d), the problem states that \( m = \frac{1}{2} \). Remind students of the meaning of the growth in relation to the pattern. This would mean every 2 figures grow by 1 tile.

**Team Strategies:** Reminding the class of their team-role responsibilities can help students work together to consolidate their knowledge. Use the General Team Role Resource Page as a prompt, or simply verbally remind different team members of their responsibilities. Specifically, remind all students that the goal at the end of this lesson is for each team member individually to be confident of moving from each representation to every other representation directly. To accomplish this, students will need to remember that helping is not the same as giving answers. Instead, students will need to ask questions, give hints, and explain with reasons.

**Homework:** Problems 4-67 through 4-71
Note: Problem 4-71 is Checkpoint 4 for area and perimeter of circles and complex figures.
4.1.7 What are the connections?

Completing the Web

After all of the work you have done with equations in \( y = mx + b \) form, you know a lot about starting with one representation of a pattern and moving to different representations. Today you will work with your team to make sure you are confident moving around the Representations of Patterns Web.

Answer problems 4-64 and 4-65 on graph paper. Discuss each problem with your team to get as much as you can out of these problems.

4-64. **GRAPH \( \rightarrow \) PATTERN and TABLE \( \rightarrow \) PATTERN**

On graph paper, draw tile patterns (Figures 0, 1, and 2) that could represent the data shown below. Be creative, but make sure that the growth of each pattern makes sense to your teammates. \[ a: y = 3x + 2, \quad b: y = -3x + 14 \]

\[
\begin{array}{c|c|c}
\hline
x & y \\
\hline
0 & 14 \\
1 & 11 \\
2 & 8 \\
3 & 5 \\
4 & 2 \\
\hline
\end{array}
\]
4-65. **REVISITING “GROWING, GROWING, GROWING”**

Problem 1-10 from Chapter 1 asked you to determine which figure in the pattern shown below would have 79 tiles. Now that you know more about tile patterns, \(x\rightarrow y\) tables, graphs, and rules, you can show the answer to this question in multiple ways.

Your Task: Solve this problem by completing the following tasks. Use a graphing calculator or other graphing technology to help you find a graph and a table. Be sure to record your work and justify your thinking.

1. Copy the three figures above onto a piece of graph paper. On your graph paper, extend the pattern to include Figures 1 and 5.
2. Find a table, a graph, and a rule, for this pattern.
3. Which figure will have 79 tiles? Use as many representations as you can to justify your answer. [Figure 19]

4-66. **EXTENSION**

Invent an equation to fit these clues: The \(x\)-intercept is 2, and the pattern grows by 4. Show and explain your reasoning. [\( y = 4x - 8 \)]
Consider the tile patterns below. The number of tiles in each figure can also be represented in an $x\rightarrow y$ table, on a graph, or with a rule (equation).

Remember that in this course, tile patterns will be considered to be elements of continuous relationships and thus will be graphed with a continuous line or curve.

<table>
<thead>
<tr>
<th>Figure Number ($x$)</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Tiles ($y$)</td>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

$y = 2x + 3$

Rule (Equation)
4-67. Use what you know about $m$ and $b$ to graph each equation below without making a table. Show a growth triangle on each graph and label the $x$- and $y$-intercepts.

[ See graphs below. a: $x: (1.5, 0), y: (0, 3)$, b: $x$ and $y: (0, 0)$, c: No $x$-intercept, $y: (0, 3)$, d: $x: (6, 0), y: (0, 3)$ ]

a. $y = 3 - 2x$

![Graph a](image)

b. $y = 2x$

![Graph b](image)

c. $y = 3$

![Graph c](image)

d. $y = \frac{-1}{2}x + 3$

![Graph d](image)

4-68. On your paper, copy and complete each $x\rightarrow y$ table below. Using what you know about $m$ and $b$, write an equation that represents the data in the table. [ See bold answers in table below. ]

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>x</td>
<td>y</td>
<td>b.</td>
<td>x</td>
<td>y</td>
<td>c.</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>−2</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>2</td>
<td>−2</td>
<td>1</td>
<td>−2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>−5</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>13</td>
<td>4</td>
<td>−4</td>
<td>3</td>
<td>−8</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>65</td>
<td>30</td>
<td>−56</td>
<td>100</td>
<td>−299</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>405</td>
<td>150</td>
<td>−296</td>
<td>100</td>
<td>−299</td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>505</td>
<td>300</td>
<td>−596</td>
<td>−23</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>x</td>
<td>$2x + 5$</td>
<td>x</td>
<td>$−2x + 4$</td>
<td>x</td>
<td>$−3x + 1$</td>
<td></td>
</tr>
</tbody>
</table>

4-69. For a tile pattern with the rule $y = 6x + 4$ (where $x$ represents the figure number and $y$ represents the number of tiles), which figure number has 40 tiles in it? How do you know? [ Figure 6. $40 = 6x + 4$ ]
4-70. Josie and Jules are building a model car. They find that the real car is 54 inches tall and 180 inches long. They decide to make their model 3 inches tall, but now they are having a disagreement. Josie thinks that their model should be 10 inches long and Jules thinks it should be 129 inches long. Help them settle their argument by deciding if either of them is correct. Explain how you know exactly how long their model should be.
[ Josie is correct, 10 inches long. ]

4-71. This problem is a checkpoint for area and perimeter of circles and complex figures. It will be referred to as Checkpoint 4.

Find the area and perimeter or circumference of each figure.

a. Circle with radius 3 cm.
   \[ A \approx 28.27 \text{ cm}^2, \quad C \approx 18.85 \text{ cm} \]
b. Circle with diameter 10 feet.
   \[ A \approx 78.54 \text{ ft}^2, \quad C \approx 31.42 \text{ ft} \]
c. Only the shaded region
   (each sector has equal area).
   \[ A \approx 150.80 \text{ ft}^2, \quad P \approx 49.13 \text{ ft} \]
d. \[ A \approx 452.53 \text{ ft}^2, \quad P \approx 85.13 \text{ ft} \]

Check your answers by referring to the Checkpoint 4 materials located at the back of your book.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 4 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.
Chapter 4 Closure  What have I learned?

Reflection and Synthesis

Closure Objective: Chapter closure provides an opportunity for students to reflect about what they have learned. See the Closure section of this Teacher Edition for more information about chapter closure.

Length of Activity: Varies

Materials: For teachers using Summarizing My Understanding:
- Poster making materials

⊙ Summarizing My Understanding
For this chapter’s summary activity, have students or teams choose or create problems to demonstrate the different connections in the Representations of Patterns Web. Along with the solutions to their problems, you can ask students to explain the process involved. If you want students to give an example for every connection on the web, they will need one of each of the following:

- graph → table
- graph → pattern
- graph → rule
- table → graph
- table → pattern
- table → rule
- pattern → graph
- pattern → table
- pattern → rule
- rule → graph
- rule → table
- rule → pattern

This could be done by having students work together in their teams to create a large Representation of Patterns Team Poster that demonstrates the all of the representations on a large copy of the web, as described in the student text. Students will need to work cooperatively and collaboratively to determine the best way to lay out their work the poster so that it is neat and presentable. Another option is to have students create an individual portfolio of sorts with each problem on a separate page with a table of contents and/or title page that is the Representation of Patterns Web. Recognize that this will require a great deal more work on the part of each student.

⊕ What Have I Learned?
This section gives students the opportunity to see if they can work with the current topics at the expected level.

⊕ What Tools Can I Use?
See the Closure section of this Teacher Edition for more information about What Tools Can I Use?.

Notes to Self:
Chapter 4 Closure  What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with.

1  SUMMARIZING MY UNDERSTANDING

This section gives you an opportunity to show your understanding of how to move between the different representations of a pattern (situation, table, graph, and rule), which is the main idea of this chapter.

Team Representations Poster

In this chapter, you have learned how to move directly from one representation to another. Now you have an opportunity to demonstrate what you know about these concepts. Today you and your team will create a poster that illustrates the skills and knowledge that you have developed in this area.

Make Connections:  Follow your teacher’s instructions to work with your team to find or create problems that demonstrate all of the connections between the different representations shown in the Representations of Patterns Web.

Along with the solutions to each of your problems, you will need to explain the process involved. To give an example for every connection on the web, you will need one of each of the following:

- graph → table
- table → graph
- pattern → graph
- rule → graph
- graph → pattern
- table → pattern
- pattern → table
- rule → table
- graph → rule
- table → rule
- pattern → rule
- rule → pattern

Team Poster:  Follow the model above to label and construct the sections of your poster from the pieces that your team has created. Decide together on a creative title for your poster.
WHAT HAVE I LEARNED?

Doing the problems in this section will help you to evaluate which types of problems you feel comfortable with and which ones you need more help with.

Solve each problem as completely as you can. The table at the end of this closure section provides answers to these problems. It also tells you where you can find additional help and where to find practice problems like them.

CL 4-72. Examine the pattern below, and then complete parts (a) through (f) that follow.

a. On graph paper, sketch Figure 0 and Figure 4.
b. Make a table showing Figure 0 through Figure 4.
c. Write a rule to represent the pattern.
d. On graph paper, create a graph of the number of tiles in each figure.
e. What is the growth for the pattern?
f. Predict how many tiles Figure 100 will have.

CL 4-73. Are the two expressions below equal? Show how you know.

\[4x^2 + 2x - 5 - 3x\] and \[6x^2 - x + 3 - 2x^2 - 8\]
CL 4-74. Examine the graph at right.
   a. Give two ways you can tell that the rule 
      \( y = 2x - 3 \) does not match the graph.
   b. Make a graph that matches the rule \( y = 2x - 3 \).
   c. Find a rule that represents the graph at right.

CL 4-75. Consider the rule \( y = 5x + 7 \).
   a. How many tiles are in Figure 0?
   b. Which figure has 37 tiles?
   c. In the equation \( y = mx + b \), what do the letters \( m \) and \( b \) represent?

CL 4-76. Molly read 75 pages of the latest thriller mystery novel in 45 minutes. What is her unit rate? At the same rate, how long will it take her to read the entire 425-page novel?

CL 4-77. Solve this equation to find \( x \): \[ 2 - (3x - 4) = 2x - 9. \]

CL 4-78. Simplify the following expressions, if possible.
   a. \[ x + 4x - 3 + 3x^2 - 2x \]
   b. \[ 2x + 4y^2 - 6y^2 - 9 - x + 3x \]
   c. \[ 3x^2 + 10y - 2y^2 + 4x - 14 \]
   d. \[ 20 + 3xy - 4xy + y^2 + 10 - y^2 \]
   e. Evaluate the expressions in parts (a) and (b) above when \( x = 5 \) and \( y = -2 \).
CL 4-79. Copy and complete the table for the linear pattern below.

<table>
<thead>
<tr>
<th>IN (x)</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>OUT (y)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-2</td>
<td>3</td>
<td>8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. What is the y-intercept? What is the growth factor?

b. Find the rule for this line.

c. If the output number (y) is –52, what was the input number (x)?

CL 4-80. For the problem below, define a variable, write an equation, and solve it. Use the 5-D Process, if needed, to help you set up your equation.

For the school play, the advance tickets cost $3, while tickets at the door cost $5. Thirty more tickets were sold at door than in advance, and $2630 was collected. How many of each kind of ticket were sold? Write your answer in a sentence.

CL 4-81. For each of the problems above, do the following:

• Draw a bar or number line that represents 0 to 10.

• Color or shade in a portion of the bar that represents your level of understanding and comfort with completing that problem on your own.

If any of your bars are less than a 5, choose one of those problems and complete one of the following tasks:

• Write two questions that you would like to ask about that problem.

• Brainstorm two things that you DO know about that type of problem.

If all of your bars are a 5 or above, choose one of those problems and do one of these tasks:

• Write two questions you might ask or hints you might give to a student who was stuck on the problem.

• Make a new problem that is similar and more challenging than that problem and solve it.
WHAT TOOLS CAN I USE?

You have several tools and references available to help support your learning – your teacher, your study team, your math book, and your Toolkit, to name only a few. At the end of each chapter you will have an opportunity to review your Toolkit for completeness as well as to revise or update it to better reflect your current understanding of big ideas.

The main elements of your Toolkit should be your Learning Logs, Math Notes, and the vocabulary used in this chapter. Math words that are new to this chapter appear in bold in the text. Refer to the lists provided below and follow your teacher’s instructions to revise your Toolkit, which will help make it a useful reference for you as you complete this chapter and prepare to begin the next one.

**Learning Log Entries**
- Lesson 4.1.2 – Representations of Patterns Web
- Lesson 4.1.3 – Connecting Linear Rules and Graphs
- Lesson 4.1.4 – $y = mx + b$
- Lesson 4.1.6 – Graphing Without an $x \rightarrow y$ Table

**Math Notes**
- Lesson 4.1.7 – Representations of Patterns

**Mathematical Vocabulary**

The following is a list of vocabulary found in this chapter. Some of the words you have been seen in previous chapters. The words in bold are the words new to this chapter. Make sure that you are familiar with the terms below and know what they mean. For the words you do not know, refer to the glossary or index. You might also add these words to your Toolkit so that you can reference them in the future.

<table>
<thead>
<tr>
<th>word</th>
<th>definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>graph</td>
<td>growth factor</td>
</tr>
<tr>
<td>parameter</td>
<td>rule</td>
</tr>
<tr>
<td>tile pattern</td>
<td>solution</td>
</tr>
<tr>
<td>$y$-intercept</td>
<td>$x \rightarrow y$ table</td>
</tr>
</tbody>
</table>

Core Connections, Course 3
Answers and Support for Closure Problems
What Have I Learned?

Note: MN = Math Note; LL = Learning Log

<table>
<thead>
<tr>
<th>Problem</th>
<th>Solution</th>
<th>Need Help?</th>
<th>More Practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>CL 4-72.</td>
<td><img src="image1" alt="Figure 0" /></td>
<td>Sections 3.1 and 4.1</td>
<td>Problems 4-12, 4-13, 4-15, 4-25, 4-37, 4-38, 4-42, 4-54, 4-64, and 4-65</td>
</tr>
<tr>
<td>a.</td>
<td><img src="image2" alt="Figure 4" /></td>
<td>MN: 4.1.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>LL: 4.1.2</td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Figure Number</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Number of Tiles</td>
<td>5</td>
<td>11</td>
<td>17</td>
</tr>
<tr>
<td>c.</td>
<td>$y = 6x + 5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e. Each figure has 6 more tiles than the previous figure.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f. Figure 100 will have 605 tiles.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CL 4-73.</td>
<td>Yes; $4x^2 - x - 5 = 4x^2 - x - 5$</td>
<td>MN: 2.1.3, 2.1.5, 2.1.6, and 2.1.7</td>
<td>Problems 3-32, 3-77, and 4-10</td>
</tr>
<tr>
<td>CL 4-74.</td>
<td>a. The line goes down as $x$ increases. The y-intercept is at +3.</td>
<td>Lessons 4.1.3, 4.1.4, and 4.1.6</td>
<td>Problems 4-23, 4-32, 4-38, 4-54, 4-55, 4-56, 4-59, and 4-60</td>
</tr>
<tr>
<td></td>
<td>b. See graph at right.</td>
<td>MN: 4.1.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c. $y = -2x + 3$</td>
<td>LL: 4.1.3 and 4.1.4</td>
<td></td>
</tr>
</tbody>
</table>

Chapter 4: Multiple Representations
CL 4-75.  

<table>
<thead>
<tr>
<th>Problem</th>
<th>Solution</th>
<th>Need Help?</th>
<th>More Practice</th>
</tr>
</thead>
</table>
| CL 4-75. | a. There are 7 tiles in Figure 0.  
b. Figure 6 has 37 tiles.  
c. \( m \) represents the growth factor, and \( b \) represents the number of tiles in Figure 0. | Lessons 4.1.2 and 4.1.4 \( \text{MN: 4.1.7} \)  
Lessons 4.1.2, 4.1.3, and 4.1.4 | Problems 4-14, 4-15, 4-36, 4-34, 4-57, 4-59, 4-61, and 4-67 |

CL 4-76.  

Unit rate \( = 1.67 \) pages per minute, 255 minutes  

| Nomen | Lesson 1.2.1 and 1.2.2 \( \text{MN: 1.2.1 and 1.2.2} \) \( \text{LL: 1.2.1} \) | Problems 4-18, 4-29, 4-63, and 4-70 |

CL 4-77.  

\( x = 3 \)  

| Nomen | Lesson 2.1.8 and 2.1.9 \( \text{MN: 2.1.3, 2.1.8, 2.1.9} \) \( \text{LL: 2.1.9} \) | Problems 2-75 and 2-83 |

CL 4-78.  

a. \( 3x^2 + 3x - 3 \)  
b. \( -2y^2 + 4x - 9 \)  
c. It cannot be simplified any further.  
d. \( -xy + 30 \)  
e. (a) 87; (b) 3  

| Nomen | Lesson 2.1.3 \( \text{MN: 2.1.3, 2.1.5, and 2.1.6} \) \( \text{LL: 2.1.9} \) | Problems 4-4, 4-10, and 4-48 |

CL 4-79.  

| Nomen | Lesson 4.1.4, MN: 4.1.7 | Problems 4-11, 4-33, 4-41, 4-37, and 4-68 |

| IN \((x)\) | -4 | -3 | -2 | -1 |
| OUT \((y)\) | -22 | -17 | -12 | -7 |

table continued:

| IN \((x)\) | 0 | 1 | 2 | 3 | 4 |
| OUT \((y)\) | -2 | 3 | 8 | 13 | 18 |

a. The starting value is -2. The growth factor is 5.  
b. \( y = 5x - 2 \)  
c. -10

CL 4-80.  

\( 5(x + 30) + 3x = 2630 \), 310 tickets were sold in advance, and 340 tickets were sold at the door.  

| Nomen | MN: 1.1.3 and 3.2.4 | Problems 3-7, 3-28, and 3-103 |

Core Connections, Course 3
Tile Pattern Team Challenge

Your team’s task is to create a poster showing every way you can represent the pattern below and highlighting all of the connections between the representations that you can find. For this activity, finding and showing the connections are the most important parts. Clearly presenting the connections between representations on your poster will help you convince your classmates that your description of the pattern makes sense.

Pattern Analysis:

- Extend the pattern: Draw Figures 0, 4, and 5. Then describe Figure 100. Give as much information as you can. What will it look like? How will the tiles be arranged? How many tiles will it have?
- Generalize the pattern by writing a rule that will give the number of tiles in any figure in the pattern. Show how you got your answer.
- Find the number of tiles in each figure. Record your data in a table and a graph.
- Demonstrate how the pattern grows using color, arrows, labels, and other math tools to help you show and explain. Show growth in each representation.
- What connections do you see between the different representations (graph, figures, and \( x \rightarrow y \) table)? How can you show these connections?

Presenting the Connections:

As a team, organize your work into a large poster that clearly shows each representation of your pattern, as well as a description of Figure 100. When your team presents your poster to the class, you will need to support each statement with a reason from your observations. Each team member must explain something mathematical as part of your presentation.
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Gallery Walk Directions

You will now have the opportunity to examine the posters created by the other teams. As you circulate, give written comments about the following:

- Does the poster describe growth in the table? In the pattern? In the graph? In the rule?
- Does the poster describe the pattern?
- Which connections does it show? For example, does the poster show connections between the table and graph? Between the pattern and rule? Pattern and graph?
- What suggestions do you have for this team? (Be serious, positive, and helpful!)
- What do you like about this team’s work?
Team Roles

**Resource Manager:**
- Get supplies for your team and make sure that your team cleans up.
  “I’ll take care of _______ if you will ______.”
- Make sure that everyone has shared all of their ideas and help the team decide when it needs outside help.
- Call the teacher over for team questions.
  “Does anyone have another idea? Are we ready to ask a question?”

**Facilitator:**
- Make sure your team understands the entire task before you begin.
  “Who wants to read? Does everyone understand what we need to do?”
  “What is the connection? How will it show in the graph? How will it show in the $x \rightarrow y$ table?”
- Keep your team together. Make sure everyone’s ideas are heard.
  “Does anyone see it in a different way?”

**Recorder/Reporter:**
- Help your team organize a poster with all of your results. Your poster needs to show everyone’s ideas and be well organized.
  “How can we show the growth?”
  “How can we show that connection?”

**Task Manager:**
- Be sure that your team is accomplishing the task effectively and efficiently.
- Keep track of the time and tell the team when it is time to move forward to the next part of the task.
- Make sure that all talking is within your team and is helping you accomplish the task.
  “Are we ready to move on?”
  “How can we divide the work most efficiently?”
  “We need to finish this part in 5 minutes, so we have time for…”
Lesson 4.1.2 Resource Page

Pattern Analysis

Figure 0  Figure 1  Figure 2  Figure 3  Figure 4

Tile Pattern #1

Tile Pattern #2

Tile Pattern #3

Tile Pattern #4
Lesson 4.1.3 Resource Page

Pattern Analysis

Tile Pattern #1

<table>
<thead>
<tr>
<th>Figure #</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Tiles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tile Pattern #2

<table>
<thead>
<tr>
<th>Figure #</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Tiles</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tile Pattern #3

<table>
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<tr>
<th>Figure #</th>
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<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Tiles</td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tile Pattern #4

<table>
<thead>
<tr>
<th>Figure #</th>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Tiles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>