Students have been solving equations since Algebra 1. Now they focus on what a solution means, both algebraically and graphically. By understanding the nature of solutions, students are able to solve equations in new and different ways. Their understanding also provides opportunities to solve some challenging applications. In this section they will extend their knowledge about solving one and two variable equations to solve systems with three variables.

Example 1

The graph of \( y = (x - 5)^2 - 4 \) is shown at right. Solve each of the following equations. Explain how the graph can be used to solve the equations.

a. \((x - 5)^2 - 4 = 12\)

b. \((x - 5)^2 - 4 = -3\)

c. \((x - 5)^2 = 4\)

Students can determine the correct answers through a variety of different ways. For part (a), most students would do the following:

\[
(x - 5)^2 - 4 = 12 \\
(x - 5)^2 = 16 \\
x - 5 = \pm 4 \\
x = 5 \pm 4 \\
x = 9, 1
\]

This is correct and a standard procedure. However, with the graph of the parabola provided, the student can find the solution by inspecting the graph. Since we already have the graph of \( y = (x - 5)^2 - 4 \), we can add the graph of \( y = 12 \) which is a horizontal line. These two graphs cross at two points, and the \( x \)-coordinates of these points are the solutions. The intersection points are (1, 12) and (9, 12). Therefore the solutions to the equation are \( x = 1 \) and \( x = 9 \).

We can use this method for part (b) as well. Draw the graph of \( y = -3 \) to find that the graphs intersect at (4, -3) and (6, -3). Therefore the solutions to part (b) are \( x = 4 \) and \( x = 6 \).
The equation in part (c) might look as if we cannot solve it with the graph, but we can. Granted, this is easy to solve algebraically:

\[(x - 5)^2 = 4\]
\[x - 5 = \pm 2\]
\[x = 7, 3\]

But, we want students to be looking for alternative approaches to solving problems. By looking for and exploring alternative solutions, students are expanding their repertoire for solving problems. This year they will encounter equations that can only be solved through alternative methods. By recognizing the equation in part (c) as equivalent to \((x - 5)^2 - 4 = 0\) (subtract four from both sides), we can use the graph to find where the parabola crosses the line \(y = 0\) (the \(x\)-axis). The graph tells us the solutions are \(x = 7\) and \(x = 3\).

**Example 2**

Solve the equation \(\sqrt{x + 2} = 2x + 1\) using at least two different approaches. Explain your methods and the implications of the solution(s).

Most students will begin by using algebra to solve this equation. This includes squaring both sides and solving a quadratic equation as shown at right.

\[
\begin{align*}
\sqrt{x + 2} &= 2x + 1 \\
(\sqrt{x + 2})^2 &= (2x + 1)^2 \\
x + 2 &= 4x^2 + 4x + 1 \\
4x^2 + 3x - 1 &= 0 \\
(4x - 1)(x + 1) &= 0
\end{align*}
\]
\[x = \frac{1}{4}, \quad -1\]

A problem arises if the students do not check their work. If the students substitute each \(x\) value back into the original equation, only one \(x\)-value checks: \(x = \frac{1}{4}\). This is the only solution. We can see why the other solution does not work if we use a graph to solve the equation. The graphs of \(y = \sqrt{x + 2}\) and \(y = 2x + 1\) are shown at right. Notice that the graphs only intersect at one point, namely \(x = \frac{1}{4}\). This is the only point where they are equal. The solution to the equation is this one value; the other value is called an **extraneous** solution.

Remember that a solution is a value that makes the equation true. In our original equation, this would mean that both sides of the equation would be equal for certain values of \(x\). Using the graphs, the solution is the \(x\)-value that has the same \(y\)-value for both graphs, or the point(s) at which the graphs intersect.
Example 3

Solve each system of equations below without graphing. For each one, explain what the solution (or lack thereof) tells you about the graph of the system.

a. \( y = -\frac{2}{5} x + 3 \) \hspace{1cm} b. \( y = -2(x - 2)^2 + 35 \) \hspace{1cm} c. \( y = \frac{1}{6} x^2 - \frac{34}{6} \)
\( y = \frac{3}{5} x - 2 \) \hspace{1cm} \( y = -2x + 15 \) \hspace{1cm} \( x^2 + y^2 = 25 \)

The two equations in part (a) are written in “y =” form, which makes them ideal candidates for the substitution method of solving. Since both expressions in \( x \) are equal to \( y \), we set the expressions equal to each other, and solve for \( x \).

\[
\begin{align*}
-\frac{2}{5} x + 3 &= \frac{3}{5} x - 2 \\
5 \left( -\frac{2}{5} x + 3 \right) &= 5 \left( \frac{3}{5} x - 2 \right) \\
-2x + 15 &= 3x - 10 \\
-5x &= -25 \\
x &= 5
\end{align*}
\]

We substitute this value for \( x \) back into either one of the original equations to determine the value of \( y \). Finally, we must check that the solution satisfies both equations.

\[
\begin{align*}
y &= -\frac{2}{5} x + 3, \ x = 5 \\
y &= -\frac{2}{5} (5) + 3 \\
&= -2 + 3 \\
&= 1 & \text{Solution to check: (5, 1)}
\end{align*}
\]

\[
\begin{align*}
y &= -\frac{2}{5} x + 3 \\
? &= -\frac{2}{5} (5) + 3 \\
1 &= -2 + 3 & \text{Check} \\
y &= \frac{3}{5} x - 2 \\
? &= \frac{3}{5} (5) - 2 \\
1 &= 3 - 2 & \text{Check}
\end{align*}
\]

Therefore the solution is the point (5, 1), which means that the graphs of these two lines intersect in one point, the point (5, 1).
The equations in part (b) are written in the same form so we will solve this system the same way we did in part (a).

\[ y = -2(x - 2)^2 + 35 \]
\[ y = -2x + 15 \]

\[-2(x - 2)^2 + 35 = -2x + 15 \]
\[-2x^2 - 4x + 4 = -2x - 20 \]
\[-2x^2 + 8x - 8 = -2x - 20 \]
\[-2x^2 + 10x + 12 = 0 \]
\[ x^2 - 5x - 6 = 0 \]
\[ (x - 6)(x + 1) = 0 \]
\[ x = 6, \ x = -1 \]

We substitute each \( x \)-value into either equation to find the corresponding \( y \)-value. Here we will use the simpler equation.

\[ x = 6, \ y = -2x + 15 \]
\[ y = -2(6) + 15 \]
\[ y = -12 + 15 \]
\[ y = 3 \]
\[ y = 3 \]
\[ \text{Solution: (6, 3)} \]

\[ x = -1, \ y = -2x + 15 \]
\[ y = -2(-1) + 15 \]
\[ y = 2 + 15 \]
\[ y = 17 \]
\[ \text{Solution: (-1, 17)} \]

Lastly, we need to check each point in both equations to make sure we do not have any extraneous solutions.

\[ (6, 3): \ y = -2(x - 2)^2 + 35 \]
\[ 3 = -2(6 - 2)^2 + 35 \]
\[ 3 = -2(16) + 35 \text{ Check.} \]

\[ (-1, 17): \ y = -2(x - 2)^2 + 35 \]
\[ 17 = -2(-1 - 2)^2 + 35 \]
\[ 17 = -2(9) + 35 \text{ Check.} \]

In solving these two equations with two unknowns, we found two solutions, both of which check in the original equations. This means that the graphs of the equations, a parabola and a line, intersect in exactly two distinct points.
Part (c) requires substitution to solve. We can replace $y$ in the second equation with what the first equation tells us it equals, but that will require us to solve an equation of degree four (an exponent of 4). Instead, we will first rewrite the first equation without fractions in an effort to simplify. We do this by multiplying both sides of the equation by 6.

$$
y = \frac{1}{6}x^2 - \frac{34}{6}
$$

$$
6y = x^2 - 34
$$

$$
6y + 34 = x^2
$$

Now we can replace the $x^2$ in the second equation with $6y + 34$.

$$
6y + 34 = x^2
$$

$$
x^2 + y^2 = 25
$$

$$
6y + 34 + y^2 = 25
$$

$$
y^2 + 6y + 34 = 25
$$

$$
y^2 + 6y + 9 = 0
$$

$$
(y + 3)(y + 3) = 0
$$

$$
y = -3
$$

Next we substitute this value back into either equation to find the corresponding $x$-value.

$y = -3$: $6y + 34 = x^2$

$$
6(-3) + 34 = x^2
$$

$$
-18 + 34 = x^2
$$

$$
16 = x^2
$$

$$
x = \pm 4
$$

This gives us two possible solutions: $(4, -3)$ and $(-4, -3)$. Be sure to check these points for extraneous solutions!

$(4, -3)$: $y = \frac{1}{6}x^2 - \frac{34}{6}$, \quad $-3 = \frac{1}{6}(4)^2 - \frac{34}{6} = \frac{16}{6} - \frac{34}{6} = -\frac{18}{6}$, \quad check.

$(4, -3)$: $x^2 + y^2 = 25$, \quad $(4)^2 + (-3)^2 = 16 + 9 = 25$, \quad check.

$(-4, -3)$: $y = \frac{1}{6}x^2 - \frac{34}{6}$, \quad $-3 = \frac{1}{6}(-4)^2 - \frac{34}{6} = \frac{16}{6} - \frac{34}{6} = -\frac{18}{6}$, \quad check.

$(-4, -3)$: $x^2 + y^2 = 25$, \quad $(-4)^2 + (-3)^2 = 16 + 9 = 25$, \quad check.

Since there are two points that make this system true, the graphs of this parabola and this circle intersect in only two points, $(4, -3)$ and $(-4, -3)$. 
Example 4

Jo has small containers of lemonade and lime soda. She once mixed one lemonade container with three containers of lime soda to make 17 ounces of a tasty drink. Another time, she combined five containers of lemonade with six containers of lime soda to produce 58 ounces of another splendid beverage. Given this information, how many ounces are in each small container of lemonade and lime soda?

We can solve this problem by using a system of equations. To start, we let \( x \) equal the number of ounces of lemonade in each small container, and \( y \) equal the number of ounces of lime soda in each of its small containers. We can write an equation that describes each mixture Jo created. The first mixture used one ounce of \( x \) liquid and three ounces of \( y \) liquid to produce 17 ounces. This can be represented as \( 1x + 3y = 17 \). The second mixture used five ounces of \( x \) liquid and six ounces of \( y \) liquid to produce 58 ounces. This can be represented by the equation \( 5x + 6y = 58 \). We can solve this system to find the values for \( x \) and \( y \).

\[
\begin{align*}
  x + 3y &= 17 \\
  5x + 6y &= 58
\end{align*}
\]

\[
\begin{align*}
  5(x + 3y) &= 5 \times 17 \\
  5x + 15y &= 85
\end{align*}
\]

\[
\begin{align*}
  5x + 6y &= 58 \\
  9y &= 27 \\
  y &= 3
\end{align*}
\]

\[
\begin{align*}
  x + 9 &= 17 \\
  x &= 8
\end{align*}
\]

(Note: you should check these values!) Therefore each container of Jo’s lemonade has 8 ounces, and each container of her lime soda has only 3 ounces.
Problems

Solve each of the following systems for \( x \) and \( y \). Then explain what the answer tells you about the graphs of the equations. Be sure to check your work.

1. \[ \begin{align*}
    x + y &= 11 \\
    3x - y &= 5
\end{align*} \]
2. \[ \begin{align*}
    2x - 3y &= -19 \\
    -5x + 2y &= 20
\end{align*} \]
3. \[ \begin{align*}
    15x + 10y &= 21 \\
    6x + 4y &= 11
\end{align*} \]
4. \[ \begin{align*}
    8x + 2y &= 18 \\
    -6x + y &= 14
\end{align*} \]
5. \[ \begin{align*}
    12x - 16y &= 24 \\
    y &= \frac{3}{4}x - \frac{3}{2}
\end{align*} \]
6. \[ \begin{align*}
    \frac{1}{2}x - 7y &= -15 \\
    3x - 4y &= 24
\end{align*} \]

The graph of \( y = \frac{1}{2}(x - 4)^2 + 3 \) is shown at right. Use the graph to solve each of the following equations. Explain how you get your answer.

7. \[ \frac{1}{2}(x - 4)^2 + 3 = 3 \]
8. \[ \frac{1}{2}(x - 4)^2 + 3 = 5 \]
9. \[ \frac{1}{2}(x - 4)^2 + 3 = 1 \]
10. \[ \frac{1}{2}(x - 4)^2 = 8 \]

Solve each equation below. Think about rewriting, looking inside, or undoing to simplify the process.

11. \[ 3(x - 4)^2 + 6 = 33 \]
12. \[ \frac{x}{4} + \frac{x}{5} = \frac{9x - 4}{20} \]
13. \[ 3 + \left(\frac{10 - 3x}{2}\right) = 5 \]
14. \[ -3\sqrt{2x - 1} + 4 = 10 \]

Solve each of the following systems of equations algebraically. What does the solution tell you about the graph of the system?

15. \[ \begin{align*}
    y &= -\frac{2}{3}x + 7 \\
    4x + 6y &= 42
\end{align*} \]
16. \[ \begin{align*}
    y &= (x + 1)^2 + 3 \\
    y &= 2x + 4
\end{align*} \]
17. \[ \begin{align*}
    y &= -3(x - 4)^2 - 2 \\
    y &= -\frac{4}{7}x + 4
\end{align*} \]
18. \[ \begin{align*}
    x + y &= 0 \\
    y &= (x - 4)^2 - 6
\end{align*} \]
19. Adult tickets for the *Mr. Moose’s Fantasy Show on Ice* are $6.50 while a child’s ticket is only $2.50. At Tuesday night’s performance, 435 people were in attendance. The box office brought in $1667.50 for that evening. How many of each type of ticket were sold?
20. The next math test will contain 50 questions. Some will be worth three points while the rest will be worth six points. If the test is worth 195 points, how many three-point questions are there, and how many six-point questions are there?

21. Reread Example 3 from Chapter 4 about Dudley’s water balloon fight. If you did this problem, you found that Dudley’s water balloons followed the path described by the equation \( y = -\frac{8}{125} (x - 10)^2 + \frac{72}{5} \). Suppose Dudley’s nemesis, in a mad dash to save his base from total water balloon bombardment, ran to the wall and set up his launcher at its base. Dudley’s nemesis launches his balloons to follow the path \( y = -x \left( x - \frac{189}{25} \right) \) in an effort to knock Dudley’s water bombs out of the air. Is Dudley’s nemesis successful? Explain.

### Answers

1. \((4, 7)\)  
2. \((-2, 5)\)  
3. no solution  
4. \((-\frac{1}{2}, 11)\)  
5. All numbers work.  
6. \((12, 3)\)

7. \(x = 4\). The horizontal line \( y = 3 \) crosses the parabola in only one point, at the vertex.

8. \(x = 2, x = 6\)

9. No solution. The horizontal line \( y = 1 \) does not cross the parabola.

10. \(x = 0, x = 8\). Add three to both sides to rewrite the equation as \( \frac{1}{2} (x - 4)^2 + 3 = 11 \). The horizontal line \( y = 11 \) crosses at these two points.

11. \(x = 7, x = 1\)

12. no solution

13. \(x = 2\)

14. No solution. (A square root must have a positive result.)

15. All real numbers. When graphed, these equations give the same line.

16. \((0, 4)\). The parabola and the line intersect only once.

17. No solution. This parabola and this line do not intersect.

18. \((2, -2)\) and \((5, -5)\). The line and the parabola intersect twice.

19. 145 adult tickets were sold, while 290 child tickets were sold.

20. There are 35 three-point questions and 15 six-point questions on the test.

21. By graphing we see that the nemesis’ balloon when launched at the base of the wall (the \(y\)-axis), hits the path of the Dudley’s water balloon. Therefore, if timed correctly, the nemesis is successful.
Once the students understand the notion of a solution, they can extend their understanding to inequalities and systems of inequalities. Inequalities typically have infinitely many solutions, and students learn ways to represent such solutions. For further information see the Math Notes boxes following problems 5-78 on page 243, 5-88 on page 247, and 5-96 on page 250.

Example 1

Solve each equation or inequality below. Explain what the solution for each one represents. Then explain how the equation and inequalities are related to each other.

\[
x^2 - 4x - 5 = 0
giving \\
x^2 - 4x - 5 < 0
\]

Students have many ways to solve the equation, including graphing, factoring, or using the Quadratic Formula. Most students will factor and use the zero-product property to solve as shown below.

\[
x^2 - 4x - 5 = 0
\]

\[
(x - 5)(x + 1) = 0
\]

\[
x = 5, \quad x = -1
\]

These are the only two values for \( x \) that make this equation true, \( x = 5 \) and \( x = -1 \).

The second quadratic is an inequality. To solve this we will utilize a number line to emphasize what the solution represents. When solving the equation, we found that the quadratic expression can be factored.

\[
x^2 - 4x - 5 = (x - 5)(x + 1)
\]

Using the factored form, we want to find all \( x \)-values so that \( (x - 5)(x + 1) < 0 \), or rephrasing it, where the product is negative. We begin by recording on a number line the places where the product equals zero. We found those two points in the previous part: \( x = 5 \) and \( x = -1 \). By placing these two points on the number line, they act as boundary points, dividing the number line into three sections. We choose any number in each of the sections to see if the number will make the inequality true or false. Solutions will make the inequality true. Note: We only need to check one point from each section. As one point goes, so goes the section! To start, choose the point \( x = -2 \). Substituting this into the inequality gives:

\[
(-2)^2 - 4(-2) - 5 \quad ? < 0
\]

\[
4 + 8 - 5 \quad ? < 0
\]

\[
7 \quad ? < 0
\]

False! Seven is NOT less than zero, so this section is NOT part of the solution.
Next we choose a point in the middle section. An easy value to try is \( x = 0 \).

\[
(0)^2 - 4(0) - 5 < 0
\]

\[
0 - 0 - 5 < 0
\]

\[
-5 < 0
\]

True! This section is part of the solution.

Finally, we check to see if any points in the last section make the inequality true. Try \( x = 7 \).

\[
(7)^2 - 4(7) - 5 < 0
\]

\[
49 - 28 - 5 < 0
\]

\[
16 < 0
\]

False! Therefore the solution is only the middle section, the numbers that lie between \(-1\) and 5.

We can represent this in a couple of ways. We can use symbols to write \(-1 < x < 5\). We can also represent the solution on the number line by shading the section of the number line that represents the solution of the inequality. Any point in the shaded section of the number line will make the inequality true.

The last inequality of the example has added a \( y \). We want to find all \( y \)-values greater than or equal to the quadratic expression. Having both \( x \) and \( y \) means we need to use an \( xy \)-coordinate graph. The graph of the parabola at right divides the plane into two regions: the part within the “bowl” of the parabola – the interior – and the region outside the parabola. The points on the parabola represent where \( y = x^2 - 4x - 5 \). We use a test point from one of the regions to check whether it will make the inequality true or false. As before, we are looking for the “true” region. The point \((0, 0)\) is an easy point to use.

\[
0 \geq (0)^2 - 4(0) - 5
\]

\[
0 \geq 0 - 0 - 5
\]

the \[
0 \geq -5
\]

True! Therefore the region containing the point \((0, 0)\) is the solution. This means any point chosen in this region, the “bowl” of parabola, will make the inequality true.

To illustrate that this region is the solution, we shade this region of the graph. Note: Since the inequality was “greater than or equal to,” the parabola itself is included in the solution. If the inequality had been strictly “greater than,” we would have made the curve dashed to illustrate that the parabola itself is not part of the solution.

To see how these equations and inequalities are related, examine the graph of the parabola. Where are the \( y \)-values of the parabola negative? Where are they equal to zero? The graph is negative when it dips below the \( x \)-axis, and this happens when \( x \) is between \(-1\) and 5. Solving the first inequality told you that as well. It equals zero at the points \( x = -1 \) and \( x = 5 \), which you found by solving the equation. Therefore, if we had the graph initially, we could have answered the first two parts quickly by looking at the graph.
Example 2

Han and Lea have been building jet roamers and pod racers. Each jet roamer requires one jet pack and three crystallic fuel tanks, while each pod racer requires two jet packs and four crystallic fuel tanks. Han and Lea’s suppliers can only produce 100 jet packs, and 270 fuel tanks each week, and due to manufacturing conditions, Han and Lea can build no more than 30 pod racers each week. Each jet roamer makes a profit of 1 tig (their form of currency) while each pod racer makes a profit of four tigs.

a. If Han and Lea receive an order for twelve jet roamers and twenty-two pod racers, how many of each part will they need to fill this order? If they can fill this order, how many tigs will they make?

b. Write a list of constraints, an inequality for each constraint, and sketch a graph showing all inequalities with the points of intersection labeled. How many jet roamers and pod racers should Han and Lea build to maximize their profits?

This problem is an example of a linear programming problem, and although the name might conjure up images of computer programming, these problems are not done on a computer. We solve this problem by creating a system of inequalities that, when graphed, creates a feasibility region. This region contains the solution for the number of jet roamers and pod racers Han and Lea should make to maximize their profit.

We begin by defining the variables. Let \( x \) represent the number of jet roamers Han and Lea will make, while \( y \) represents the number of pod racers. We know that \( x \geq 0, \) and \( y \geq 0. \) A jet roamer requires one jet pack while a pod racer requires two. There are only 100 jet packs available each week, so we can write \( x + 2y \leq 100 \) as one of our inequalities. Each jet roamer requires three crystallic fuel tanks and each pod racer requires four. This translates into the inequality \( 3x + 4y \leq 270 \) since they have only 270 fuel tanks available each week. Lastly, since Han and Lea cannot make more than 30 pod racers, we can write \( y \leq 30. \)

To find the number of parts needed to fill the order in part (a), we can use these inequalities with \( x = 12 \) and \( y = 22. \)

Jet packs:  
\[ 12 + 2(22) = 12 + 44 = 56 \]

Crystallic fuel tank:  
\[ 3(12) + 4(22) = 36 + 88 = 124 \]

Each result is within the constraints, so it is possible for Han and Lea to fill this order. If they do, they will make \( 4(12) + 1(22) = 48 + 22 = 70 \) tigs.

Part (b) has us generalize this information to determine how Han and Lea can maximize their profits. Given their limited supply of parts, should they use all of them to make jet roamers? They bring in more money per vehicle. Or, should they make some combination of the two vehicle types to ensure they use their parts and still bring in as much money as possible? We include all these inequalities on the graph of this system at right. The region common to all constraints is shaded. This is the feasibility region because choosing a point in this shaded area gives you a combination of jet roamers and pod racers that Han and Lea can produce under the given restraints.
To maximize profits, we will test all the vertices of this region in our profit equation, profit = x + 4y, to find the greatest profit. These points are: (0, 0), (0, 30), (40, 30), (70, 15), and (90, 0).

profit: x + 4y

(0, 0): 0 + 4(0) = 0
(0, 30): 0 + 4(30) = 120
(40, 30): 40 + 4(30) = 160
(70, 15): 70 + 4(15) = 130
(90, 0): 90 + 4(0) = 90

The greatest profit is 160 tigs when Han and Lea build 40 jet roamers and 30 pod racers.

Problems

Graph the following system of inequalities. Darken in the solution (the region satisfying all inequalities).

1. \[ y < \frac{1}{2} x + 6 \]
   \[ y > -\frac{1}{2} x + 6 \]
   \[ x < 12 \]

2. \[ x + y < 10 \]
   \[ x + y > 4 \]
   \[ y < 2x \]
   \[ y > 0 \]

3. \[ y < 3x + 4 \]
   \[ y > -\frac{1}{4} x + 8 \]
   \[ y > -\frac{1}{3} x + 4 \]
   \[ y > 5x - 6 \]

4. \[ 3x + 4y < 12 \]
   \[ y > (x + 1)^2 - 4 \]

5. \[ y < -\frac{3}{4} (x - 1)^2 + 6 \]
   \[ y > x - 7 \]

6. \[ y < (x + 2)^3 \]
   \[ y > x^2 + 3x \]

Write a system of inequalities that when graphed will produce these regions.

7.

8.
9. Ramon and Thea are considering opening their own business. They wish to make and sell alien dolls they call Hauteans and Zotions. Each Hautean sells for $1.00 while each Zotion sells for $1.50. They can make up to 20 Hauteans and 40 Zotions, but no more than 50 dolls total. When Ramon and Thea go to city hall to get a business license, they find there are a few more restrictions on their production. The number of Zotions (the more expensive item) can be no more than three times the number of Hauteans (the cheaper item). How many of each doll should Ramon and Thea make to maximize their profit? What will the profit be?

10. Sam and Emma are plant managers for the Sticky Chewy Candy Company that specializes in delectable gourmet candies. Their two most popular candies are Chocolate Chews and Peanut and Jelly Jimmies. Each batch of Chocolate Chews takes 1 teaspoon of vanilla while each batch of the Peanut and Jelly Jimmies uses two teaspoons of vanilla. They have at most 20 teaspoons of vanilla on hand as they use only the freshest of ingredients. The Chocolate Chews use two teaspoons of baking soda while the Peanut and Jelly Jimmies use three teaspoons of baking soda. They only have 36 teaspoons of baking soda on hand. Because of production restrictions, they can make no more than 15 batches of Chocolate Chews and no more than 7 batches of Peanut and Jelly Jimmies. Sam and Emma have been given the task of determining how many batches of each candy they should produce if they make $3.00 profit for each batch of Chocolate Chews and $2.00 for each batch of Peanut and Jelly Jimmies. Help them out by writing the inequalities described here, graphing the feasibility region, and determining their maximum profit.

Answers

1. 

2. 

3. 

4. 

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5.  

6.  

7.  \[ y \leq \frac{1}{3}x + 4 \]
\[ y \leq -x + 8 \]
\[ y \geq -\frac{1}{2}x + 4 \]

8.  \[ y \geq (x - 6)^2 - 5 \]
\[ y \leq 0 \]

9.  The graph of the feasibility region is shown at right. The inequalities are \( x + y \leq 50 \), \( x \leq 20 \), \( y \leq 40 \), and \( y \leq 3x \), where \( x \) is the number of Hauteans and \( y \) is the number of Zotions. The profit is given by \( P = x + 1.5y \). Maximum profit seems to occur at point A (12.5, 37.5), but there is a problem with this point. Ramon and Thea cannot make a half of a doll (or at least that does not seem possible). Try these nearby points: (12, 37), (12, 38), (13, 37), and (13, 38). The point that gives maximum profit and is still in the feasibility region is (13, 37). They should make 13 Hautean and 37 Zotion dolls for a profit of $68.50.

10. The graph of the feasibility region is shown at right. The inequalities are \( y \leq 7 \), \( x \leq 15 \), \( x + 2y \leq 20 \), \( 2x + 3y \leq 36 \) where \( x \) is the number of Chocolate Chews and \( y \) is the number of Peanut and Jelly Jimmies. The profit is given by \( P = 3x + 2y \). The point that gives the maximum profit is (12, 4) which means Sam and Emma should make 12 batches of Chocolate Chews and 4 batches of Peanut and Jelly Jimmies. Their profit will be $44.00.
CHECKPOINT PRACTICE PROBLEMS

Starting in Chapter 2, several problems are marked with the icon shown at right. This icon indicates a “Checkpoint” problem for an Algebra 1 topics that students should be able to solve correctly at this point in the course. If the student needs help to solve this kind of problem, or cannot consistently solve them correctly, then the student needs additional practice with this type of problem. After each Checkpoint, the student will be expected to solve that type of problem easily and accurately.

The Checkpoint problems for Chapter 5 are problem 5-39 (function notation and domain and range) and 5-84 (solving for $y$). The practice problems below cover only these topics.

Consider the functions $f(x) = \frac{1}{3}(x - 4)^2$ and $g(x) = \frac{7}{x-2}$ (problem 4-139).

1. What are the domain and range of $f(x)$?
2. What are the domain and range of $g(x)$?
3. What is the value of $f(-2)$?
4. What is the value of $g(a + 5)$?
5. If $f(x) = 27$, what is the value of $x$?
6. If $g(x) = 14$, what is the value of $x$?
7. What are the domain and range of the function $h(x) = 3\sqrt{2x - 1}$?
8. If $j(x) = \frac{x^2 + 2x + 1}{x+1}$ what is $j(b - 4)$?
9. If $k(x) = 5 + \sqrt{7 - x}$ and $k(x) = 8$, what is the value of $x$?

Solve for $y$. (problem 5-84)

10. $8x + y = 7$
11. $6x + 3y = 9$
12. $10x + 2(y - 4) = 18$
13. $x = (y - 2)^2 + 1$
14. $\sqrt{y + 2} = 3x$
15. $\frac{x-7}{y} + 3 = 6$
Answers

1. Domain: all real numbers, range: \( y \geq 0 \).

2. Domain: \( x \neq 2 \), range: \( y \neq 0 \).

3. 12

4. \( \frac{7}{a+3} \)

5. \( x = 13, -5 \)

6. \( x = \frac{5}{2} \)

7. Domain: \( x \geq \frac{1}{2} \), Range: \( y \geq 0 \)

8. \( b = 3 \)

9. \( x = -2 \)

10. \( y = -8x + 7 \)

11. \( y = -2x + 3 \)

12. \( y = -5x + 13 \)

13. \( y = 2 \pm \sqrt{x-1} \)

14. \( y = 9x^2 - 2 \)

15. \( y = \frac{x-7}{3} \)
SAT PREP

1. If \( \frac{x+4}{12} = \frac{4}{3} \), then \( x \) equals
   a. 3   b. 6   c. 8   d. 10   e. 12

2. What is the least of three consecutive integers whose sum is 21?
   a. 5   b. 6   c. 7   d. 8   e. 9

3. Juanita has stocks, bonds, and t-bills for investments. The number of t-bills she has is one more than the number of stocks, and the number of bonds is three times the number of t-bills. Which of the following could be the total number of investments?
   a. 16   b. 17   c. 18   d. 19   e. 20

4. Through how many degrees would the minute hand of a clock turn from 5:20 p.m. to 5:35 p.m. the same day?
   a. 15°   b. 30°   c. 45°   d. 60°   e. 90°

5. The length of a rectangle is six times its width. If the perimeter of the rectangle is 56, what is the width of the rectangle?
   a. 4   b. 7   c. 8.5   d. 18   e. 24

6. If \( m > 1 \) and \( m^n m^5 = m^{15} \), then what does \( n \) equal?

7. In the triangle at right, what is the value of \( a + b + c + d \)?

8. If \( x \) and \( y \) are positive integers, \( x + y < 12 \), and \( x > 4 \), what is the greatest possible value for \( x - y \)?

9. If \( (2x^2 + 5x + 3)(2x + 4) = ax^3 + bx^2 + cx + d \) for all values of \( x \) what does \( c \) equal?

10. Four lines intersect in one point creating eight congruent adjacent angles. What is the measure of one of these angles?
Answers
1. E
2. B
3. D
4. E
5. A
6. 10
7. 280°
8. 9
9. 26
10. 45°