Multiplying Fractions with an Area Model  5.1.1, 5.1.4, 5.2.2

Multiplication of fractions is reviewed using a rectangular area model. Lines that divide the rectangle to represent one fraction are drawn vertically, and the correct number of parts are shaded. Then lines that divide the rectangle to represent the second fraction are drawn horizontally and part of the shaded region is darkened to represent the product of the two fractions.

Example 1

\[ \frac{1}{2} \cdot \frac{5}{8} \] (that is, \( \frac{1}{2} \) of \( \frac{5}{8} \))

Step 1: Draw a generic rectangle and divide it into 8 pieces vertically. Lightly shade 5 of those pieces. Label it \( \frac{5}{8} \).

Step 2: Use a horizontal line and divide the generic rectangle in half. Darkly shade \( \frac{1}{2} \) of \( \frac{5}{8} \) and label it.

Step 3: Write a number sentence. \[ \frac{1}{2} \cdot \frac{5}{8} = \frac{5}{16} \]

The rule for multiplying fractions derived from the models above is to multiply the numerators, then multiply the denominators. Simplify the product when possible.

For additional information, see the Math Notes box in Lesson 5.1.4 of the Core Connections, Course 1 text. For additional examples and practice, see the Core Connections, Course 1 Checkpoint 7A materials.

Example 2

a. \[ \frac{2}{3} \cdot \frac{2}{7} \Rightarrow \frac{2 \cdot 2}{3 \cdot 7} \Rightarrow \frac{4}{21} \]

b. \[ \frac{3}{4} \cdot \frac{6}{7} \Rightarrow \frac{3 \cdot 6}{4 \cdot 7} \Rightarrow \frac{18}{28} \Rightarrow \frac{9}{14} \]
Problems

Draw an area model for each of the following multiplication problems and write the answer.

1. \( \frac{1}{3} \cdot \frac{1}{6} \)
2. \( \frac{1}{4} \cdot \frac{3}{5} \)
3. \( \frac{2}{3} \cdot \frac{5}{9} \)

Use the rule for multiplying fractions to find the answer for the following problems. Simplify when possible.

4. \( \frac{1}{3} \cdot \frac{2}{5} \)

5. \( \frac{2}{3} \cdot \frac{2}{7} \)

6. \( \frac{3}{4} \cdot \frac{1}{5} \)

7. \( \frac{2}{5} \cdot \frac{2}{3} \)

8. \( \frac{2}{3} \cdot \frac{1}{4} \)

9. \( \frac{5}{6} \cdot \frac{2}{3} \)

10. \( \frac{4}{5} \cdot \frac{3}{4} \)

11. \( \frac{2}{15} \cdot \frac{1}{2} \)

12. \( \frac{3}{7} \cdot \frac{1}{2} \)

13. \( \frac{3}{8} \cdot \frac{4}{5} \)

14. \( \frac{2}{9} \cdot \frac{3}{5} \)

15. \( \frac{3}{10} \cdot \frac{5}{7} \)

16. \( \frac{5}{11} \cdot \frac{6}{7} \)

17. \( \frac{5}{6} \cdot \frac{3}{10} \)

18. \( \frac{10}{11} \cdot \frac{3}{5} \)

19. \( \frac{5}{12} \cdot \frac{3}{5} \)

20. \( \frac{7}{9} \cdot \frac{5}{14} \)

Answers

1. \( \frac{1}{18} \)

2. \( \frac{3}{20} \)

3. \( \frac{10}{27} \)

4. \( \frac{2}{15} \)

5. \( \frac{4}{21} \)

6. \( \frac{3}{20} \)

7. \( \frac{4}{15} \)

8. \( \frac{2}{12} = \frac{1}{6} \)

9. \( \frac{10}{18} = \frac{5}{9} \)

10. \( \frac{12}{20} = \frac{3}{5} \)

11. \( \frac{2}{30} = \frac{1}{15} \)

12. \( \frac{3}{14} \)

13. \( \frac{12}{40} = \frac{3}{10} \)

14. \( \frac{6}{48} = \frac{2}{16} \)

15. \( \frac{15}{70} = \frac{3}{14} \)

16. \( \frac{30}{77} \)

17. \( \frac{15}{60} = \frac{1}{4} \)

18. \( \frac{30}{55} = \frac{6}{11} \)

19. \( \frac{15}{60} = \frac{1}{4} \)

20. \( \frac{35}{126} = \frac{5}{18} \)
MULTIPLYING DECIMALS AND PERCENTS

Understanding how many places to move the decimal point in a decimal multiplication problem is connected to the multiplication of fractions and place value.

Computations involving calculating “a percent of a number” are simplified by changing the percent to a decimal.

Example 1

Multiply $(0.2) \cdot (0.3)$.

In fractions this means $\frac{2}{10} \cdot \frac{3}{10} \Rightarrow \frac{6}{100}$.

Knowing that the answer must be in the hundredths place tells you how many places to move the decimal point (to the left) without using the fractions.

$(\text{tenths})(\text{tenths}) = \text{hundredths}$

Therefore move two places.

\[
\begin{array}{c}
0.2 \\
\times 0.3 \\
\hline
0.06
\end{array}
\]

Example 2

Multiply $(1.7) \cdot (0.03)$.

In fractions this means $\frac{17}{10} \cdot \frac{3}{100} \Rightarrow \frac{51}{1000}$.

Knowing that the answer must be in the thousandths place tells you how many places to move the decimal point (to the left) without using the fractions.

$(\text{tenths})(\text{hundredths}) = \text{thousandths}$

Therefore move three places.

\[
\begin{array}{c}
1.7 \\
\times 0.03 \\
\hline
0.051
\end{array}
\]

Example 3

Calculate $17\%$ of $32.5$ without using a calculator.

Since $17\% = \frac{17}{100} = 0.17$,

$17\%$ of $32.5 \Rightarrow (0.17) \cdot (32.5)$

$\Rightarrow 5.525$
Problems

Identify the number of places to the left to move the decimal point in the product. Do not compute the product.

1. \((0.3) \cdot (0.5)\)  
2. \((1.5) \cdot (0.12)\)  
3. \((1.23) \cdot (2.6)\)  
4. \((0.126) \cdot (3.4)\)  
5. \(17 \cdot (32.016)\)  
6. \((4.32) \cdot (3.1416)\)

Compute without using a calculator.

7. \((0.8) \cdot (0.03)\)  
8. \((3.2) \cdot (0.3)\)  
9. \((1.75) \cdot (0.09)\)  
10. \((4.5) \cdot (3.2)\)  
11. \((1.8) \cdot (0.032)\)  
12. \((7.89) \cdot (6.3)\)  
13. \(8\%\) of 540  
14. \(70\%\) of 478  
15. \(37\%\) of 4.7  
16. \(17\%\) of 96  
17. \(15\%\) of 4.75  
18. \(130\%\) of 42

Answers

1. 2  
2. 3  
3. 3  
4. 4  
5. 3  
6. 6  
7. 0.024  
8. 0.96  
9. 0.1575  
10. 14.4  
11. 0.0576  
12. 49.707  
13. 43.2  
14. 334.6  
15. 1.739  
16. 16.32  
17. 0.7125  
18. 54.6
Area is the number of non-overlapping square units needed to cover the interior region of a two-dimensional figure or the surface area of a three-dimensional figure. For example, area is the region that is covered by floor tile (two-dimensional) or paint on a box or a ball (three-dimensional).

For additional information about specific shapes, see the boxes below.

### AREA OF A RECTANGLE

To find the area of a rectangle, follow the steps below.

1. Identify the base.
2. Identify the height.
3. Multiply the base times the height to find the area in square units: $A = bh$.

A square is a rectangle in which the base and height are of equal length. Find the area of a square by multiplying the base times itself: $A = b^2$.

**Example**

<table>
<thead>
<tr>
<th>base = 8 units</th>
<th>height = 4 units</th>
</tr>
</thead>
<tbody>
<tr>
<td>32 square units</td>
<td></td>
</tr>
</tbody>
</table>

$A = 8 \cdot 4 = 32$ square units
Problems

Find the areas of the rectangles (figures 1-8) and squares (figures 9-12) below.

1. \( \text{2 mi} \times \text{4 mi} \)

2. \( \text{5 cm} \times \text{6 cm} \)

3. \( \text{7 in.} \times \text{3 in.} \)

4. \( \text{8 m} \times \text{2 m} \)

5. \( \text{5.5 miles} \times \text{2 miles} \)

6. \( \text{3 units} \times \text{8.7 units} \)

7. \( \text{6.8 cm} \times \text{3.5 cm} \)

8. \( \text{7.25 miles} \times \text{2.2 miles} \)

9. \( \text{8 cm} \times \text{8 cm} \)

10. \( \text{2.2 cm} \times \text{2.2 cm} \)

11. \( \text{1.5 feet} \times \text{1.5 feet} \)

12. \( \text{8.61 feet} \times \text{8.61 feet} \)

Answers

1. 8 sq. miles
2. 30 sq. cm
3. 21 sq. in.
4. 16 sq. m
5. 11 sq. miles
6. 26.1 sq. feet
7. 23.8 sq. cm
8. 15.95 sq. miles
9. 64 sq. cm
10. 4.84 sq. cm
11. 2.25 sq. feet
12. 73.96 sq. feet
**AREA OF A PARALLELOGRAM**

A parallelogram is easily changed to a rectangle by separating a triangle from one end of the parallelogram and moving it to the other end as shown in the three figures below. For additional information, see the Math Notes box in Lesson 5.3.3 of the *Core Connections, Course 1* text.

<table>
<thead>
<tr>
<th>parallelogram</th>
<th>move triangle</th>
<th>rectangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>Step 2</td>
<td>Step 3</td>
</tr>
</tbody>
</table>

To find the area of a parallelogram, multiply the base times the height as you did with the rectangle: \( A = bh \).

**Example**

![Parallelogram](image)

- Base = 9 cm
- Height = 6 cm
- \( A = 9 \cdot 6 = 54 \) square cm

**Problems**

Find the area of each parallelogram below.

1.

![Parallelogram](image)

- Base = 8 feet
- Height = 6 feet

2.

![Parallelogram](image)

- Base = 10 cm
- Height = 8 cm

3.

![Parallelogram](image)

- Base = 11 m
- Height = 4 m

4.

![Parallelogram](image)

- Base = 13 cm
- Height = 3 cm

5.

![Parallelogram](image)

- Base = 12 in.
- Height = 7.5 in.

6.

![Parallelogram](image)

- Base = 15 ft
- Height = 11.2 ft

7.

![Parallelogram](image)

- Base = 11.3 cm
- Height = 9.8 cm

8.

![Parallelogram](image)

- Base = 15.7 cm
- Height = 8.4 cm

**Answers**

1. 48 sq. feet  
2. 80 sq. cm  
3. 44 sq. m  
4. 39 sq. cm  
5. 90 sq. in.  
6. 168 sq. ft  
7. 110.74 sq. cm  
8. 131.88 sq. cm
AREA OF A TRIANGLE

The area of a triangle is equal to one-half the area of a parallelogram. This fact can easily be shown by cutting a parallelogram in half along a diagonal (see below). For additional information, see Math Notes box in Lesson 5.3.4 of the Core Connections, Course 1 text.

As you match the triangles by either cutting the parallelogram apart or by folding along the diagonal, the result is two congruent (same size and shape) triangles. Thus, the area of a triangle has half the area of the parallelogram that can be created from two copies of the triangle.

To find the area of a triangle, follow the steps below.
1. Identify the base.
2. Identify the height.
3. Multiply the base times the height.
4. Divide the product of the base times the height by 2: \( A = \frac{bh}{2} \) or \( \frac{1}{2}bh \).

**Example 1**

- base = 16 cm
- height = 8 cm

\[
A = \frac{16 \times 8}{2} = \frac{128}{2} = 64 \text{ cm}^2
\]

**Example 2**

- base = 7 cm
- height = 4 cm

\[
A = \frac{7 \times 4}{2} = \frac{28}{2} = 14 \text{ cm}^2
\]
Problems

1. 
   \[
   \begin{array}{c}
   6 \text{ cm} \\
   \hline
   8 \text{ cm}
   \end{array}
   \]

2. 
   \[
   \begin{array}{c}
   12 \text{ ft} \\
   \hline
   14 \text{ ft}
   \end{array}
   \]

3. 
   \[
   \begin{array}{c}
   13 \text{ cm} \\
   \hline
   6 \text{ cm}
   \end{array}
   \]

4. 
   \[
   \begin{array}{c}
   8 \text{ in.} \\
   \hline
   17 \text{ in.}
   \end{array}
   \]

5. 
   \[
   \begin{array}{c}
   5 \text{ ft} \\
   \hline
   7 \text{ ft}
   \end{array}
   \]

6. 
   \[
   \begin{array}{c}
   1.5 \text{ m} \\
   \hline
   5 \text{ m}
   \end{array}
   \]

7. 
   \[
   \begin{array}{c}
   9 \text{ cm} \\
   \hline
   21 \text{ cm}
   \end{array}
   \]

8. 
   \[
   \begin{array}{c}
   7 \text{ ft} \\
   \hline
   2.5 \text{ ft}
   \end{array}
   \]

Answers

1. 24 sq. cm
2. 84 sq. ft
3. 39 sq. cm
4. 68 sq. in.
5. 17.5 sq. ft
6. 3.75 sq. m
7. 94.5 sq. cm
8. 8.75 sq. ft
**AREA OF A TRAPEZOID**

A trapezoid is another shape that can be transformed into a parallelogram. Change a trapezoid into a parallelogram by following the three steps below.

1. **Step 1**
   - Duplicate the trapezoid and rotate.
2. **Step 2**
   - Put the two trapezoids together to form a parallelogram.
3. **Step 3**
   - To find the area of a trapezoid, multiply the base of the large parallelogram (base and top) times the height and then take half of the total area. Remember to add the lengths of the base and the top of the trapezoid before multiplying by the height. Note that some texts call the top length the upper base and the base the lower base.

\[
A = \frac{1}{2} (b + t)h \quad \text{or} \quad A = \frac{b + t}{2} \cdot h
\]

For additional information, see the Math Notes box in Lesson 6.1.1 of the *Core Connections, Course 1* text.

**Example**

```
8 in.  4 in.  
\[ \text{top} = 8 \text{ in.} \]
\[ \text{base} = 12 \text{ in.} \]
\[ \text{height} = 4 \text{ in.} \]
```

\[
A = \frac{8+12}{2} \cdot 4 = \frac{20}{2} \cdot 4 = 10 \cdot 4 = 40 \text{ in.}^2
\]
Problems

Find the areas of the trapezoids below.

1. \[ \frac{1}{2} \times (3 + 5) \times 1 = 4 \text{ sq. cm} \]

2. \[ \frac{1}{2} \times (10 + 15) \times 8 = 100 \text{ sq. in.} \]

3. \[ \frac{1}{2} \times (2 \times 5) = \text{area not shown} \]

4. \[ \frac{1}{2} \times (11 + 15) \times 8 = 104 \text{ sq. cm} \]

5. \[ \frac{1}{2} \times (7 + 10) \times 5 = 42.5 \text{ sq. in.} \]

6. \[ \frac{1}{2} \times (11 + 8) \times 8 = 76 \text{ sq. m} \]

7. \[ \frac{1}{2} \times (7 + 10.5) \times 4 = 35 \text{ sq. cm} \]

8. \[ \frac{1}{2} \times (8.4 + 6.5) \times 3 = 22.35 \text{ sq. cm} \]

Answers

1. 4 sq. cm
2. 100 sq. in.
3. 14 sq. feet
4. 104 sq. cm
5. 42.5 sq. in.
6. 76 sq. m
7. 35 sq. cm
8. 22.35 sq. cm.
DIVISION BY FRACTIONS

Division by fractions introduces three methods to help students understand how dividing by fractions works. In general, think of division for a problem like $8 \div 2$ as, “In 8, how many groups of 2 are there?” Similarly, $\frac{1}{2} \div \frac{1}{4}$ means, “In $\frac{1}{2}$, how many fourths are there?”

For more information, see the Math Notes boxes in Lessons 7.2.2 and 7.2.4 of the Core Connections, Course 1 text. For additional examples and practice, see the Core Connections, Course 1 Checkpoint 8B materials. The first two examples show how to divide fractions using a diagram.

**Example 1**

Use the rectangular model to divide: $\frac{1}{2} \div \frac{1}{4}$.

Step 1: Using the rectangle, we first divide it into 2 equal pieces. Each piece represents $\frac{1}{2}$. Shade $\frac{1}{2}$ of it.

Step 2: Then divide the original rectangle into four equal pieces. Each section represents $\frac{1}{4}$. In the shaded section, $\frac{1}{2}$, there are 2 fourths.

Step 3: Write the equation. $\frac{1}{2} + \frac{1}{4} = 2$

**Example 2**

In $\frac{3}{4}$, how many $\frac{1}{2}$’s are there? 
That is, $\frac{3}{4} \div \frac{1}{2} = ?$

In $\frac{3}{4}$ there is one full $\frac{1}{2}$ shaded and half of another one (that is half of one-half).

So: $\frac{3}{4} + \frac{1}{2} = 1 \frac{1}{2}$
(one and one-half halves)