Answers

1. 19  
2. 5  
3. 70  
4. 64  
5. 62  
6. 30  
7. 9  
8. 39  
9. 12  
10. 0  
11. 54  
12. 109  
13. 44  
14. 5  
15. 47  
16. –5  
17. –25  
18. –6  
19. –10

PERIMETER USING ALGEBRA TILES  
3.1.1 and 3.1.2

Algebraic expressions can be represented by the perimeters of algebra tiles (rectangles and squares) and combinations of algebra tiles. The dimensions of each tile are shown along its sides and the tile is named by its area as shown on the tile itself in the figures at right. When using the tiles, perimeter is the distance around the exterior of a figure. For additional information, see the Math Notes boxes on pages 156 and 161 of the text.

Example 1

\[ P = 6x + 4 \text{ units} \]

Example 2

\[ P = 6x + 8 \text{ units} \]
Problems

Determine the perimeter of each figure.

1.  
2.  
3.  
4.  
5.  
6.  
7.  
8.  

Answers

1. $4x + 4$ un.  
2. $4x + 4$ un.  
3. $2x + 8$ un.  
4. $4x + 6$ un.  
5. $4x + 4$ un.  
6. $4x + 2$ un.  
7. $4x + 4$ un.  
8. $2x + 4$ un.

COMBINING LIKE TERMS 3.1.3

Algebraic expressions can also be simplified by combining (adding or subtracting) terms that have the same variable(s) into one term. Note that "same variable" means that the variable and its exponent are the same. The skill of combining like terms is necessary for solving equations. For additional information, see the Math Notes box on page 161 of the text.

Example 1

Combine like terms to simplify the expression $3x + 5x + 7x$.

All these terms have $x$ as the variable, so they are combined into one term, $15x$. 

Parent Guide
Example 2

Simplify the expression \(3x + 12 + 7x + 5\).

The terms with \(x\) can be combined. The terms without variables (the constants) can also be combined.

\[
\begin{align*}
3x + 12 + 7x + 5 & \rightarrow 3x + 7x + 12 + 5 \\
10x + 17 & \rightarrow \text{Note that in the simplified form the term with the variable is listed before the constant term.}
\end{align*}
\]

Example 3

Simplify the expression \(5x + 4x^2 + 10 + 2x^2 + 2x - 6 + x - 1\).

\[
\begin{align*}
5x + 4x^2 + 10 + 2x^2 + 2x - 6 + x - 1 & \rightarrow 4x^2 + 2x^2 + 5x + 2x + x + 10 - 6 - 1 \\
6x^2 + 8x + 3 & \rightarrow \text{Note that terms with the same variable but with different exponents are not combined and are listed in order of decreasing power of the variable, in simplified form, with the constant term last.}
\end{align*}
\]

Example 4

The algebra tiles, as shown in the previous section, are used to model how to combine like terms.

The large square represents \(x^2\), the rectangle represents \(x\), and the small square represents one. We can only combine tiles that are alike: large squares with large squares, rectangles with rectangles, and small squares with small square. If we have to combine:

\[
2x^2 + 3x + 4 \text{ and } 3x^2 + 5x + 7
\]

visualize the tiles to help combine the like terms:

\[
\begin{align*}
2x^2 (2 \text{ large squares}) + 3x (3 \text{ rectangles}) + 4 \text{ (4 small squares)} \\
+ 3x^2 (3 \text{ large squares}) + 5x (5 \text{ rectangles}) + 7 \text{ (7 small squares)}
\end{align*}
\]

The combination of the two sets of tiles, written algebraically, is: \(5x^2 + 8x + 11\).
Example 5

Sometimes it is helpful to take an expression that is written horizontally, circle the terms with their signs, and rewrite like terms in vertical columns before you combine them:

\[(2x^2 - 5x + 6) + (3x^2 + 4x - 9)\]

This procedure may make it easier to identify the terms as well as the sign of each term.

Problems

Combine the following sets of terms.

1. \((2x^2 + 6x + 10) + (4x^2 + 2x + 3)\)
2. \((3x^2 + x + 4) + (x^2 + 4x + 7)\)
3. \((8x^2 + 3) + (4x^2 + 5x + 4)\)
4. \((4x^2 + 6x + 5) - (3x^2 + 2x + 4)\)
5. \((4x^2 - 7x + 3) + (2x^2 - 2x - 5)\)
6. \((3x^2 - 7x) - (x^2 + 3x - 9)\)
7. \((5x + 6) + (-5x^2 + 6x - 2)\)
8. \(2x^2 + 3x + x^2 + 4x - 3x^2 + 2\)
9. \(3c^2 + 4c + 7x - 12 + (-4c^2) + 9 - 6x\)
10. \(2a^2 + 3a^3 - 4a^2 + 6a + 12 - 4a + 2\)

Answers

1. \(6x^2 + 8x + 13\)
2. \(4x^2 + 5x + 11\)
3. \(12x^2 + 5x + 7\)
4. \(x^2 + 4x + 1\)
5. \(6x^2 - 9x - 2\)
6. \(2x^2 - 10x + 9\)
7. \(-5x^2 + 11x + 4\)
8. \(7x + 2\)
9. \(-c^2 + 4c + x - 3\)
10. \(3a^3 - 2a^2 + 2a + 14\)
The 5-D process is one method that students can use to solve various types of problems, especially word problems. The D’s stand for Describe, Define, Do, Decide, and Declare. When students use the 5-D process, it provides a record of the student’s thinking. The patterns in the table lead directly to writing algebraic equations for the word problems.

Writing equations is one of the most important algebra skills students learn. Using the 5-D process helps to make this skill accessible to all students. In order to help students see the relationships in a word problem, we require them to include at least four entries (rows) in their tables. The repetition of the operations is needed to see how the columns are related. After students have had practice using the 5-D process to solve problems, we begin generalizing from the patterns in the table to write an equation that represents the relationships in the problem.

We also believe that writing the answer in a sentence after the table is complete is important because many students forget what the question actually was. The sentence helps the student see the “big picture” and brings closure to the problem.

See the Math Notes box on page 179 of the text for a detailed, step-by-step demonstration of a word problem similar to the one below that is solved using the 5-D process.

Example 1

A box of fruit has three times as many nectarines as grapefruit. Together there are 36 pieces of fruit. How many pieces of each type of fruit are there?

Step 1: **Describe:** Number of nectarines is three times the number of grapefruit.
Number of nectarines plus number of grapefruit equals 36.

Step 2: **Define:** Set up a table with columns. The first column should be the item you know the least about. Choose any easy amount for that column.

<table>
<thead>
<tr>
<th>Define</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Grapefruit</td>
</tr>
<tr>
<td>Trial 1:</td>
</tr>
</tbody>
</table>

What else do we need to know?

The number of nectarines, which is three times the number of grapefruit.
Example continued from previous page.

Step 3:  **Do:** What is the total number of fruit?

<table>
<thead>
<tr>
<th>Define</th>
<th>Do</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Grapefruit</td>
<td># of Nectarines</td>
</tr>
<tr>
<td>Trial 1:</td>
<td>11</td>
</tr>
</tbody>
</table>

Step 4:  **Decide:** We need to check the total pieces of fruit based on trial #1 of 11 grapefruit and compare it to the total given in the problem.

<table>
<thead>
<tr>
<th>Define</th>
<th>Do</th>
<th>Decide</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Grapefruit</td>
<td># of Nectarines</td>
<td>Total Pieces of Fruit</td>
</tr>
<tr>
<td>Trial 1:</td>
<td>11</td>
<td>33</td>
</tr>
</tbody>
</table>

Start another trial. Our total was 44; the total needed is 36, so our trial started too high and our next trial should start lower.

<table>
<thead>
<tr>
<th>Define</th>
<th>Do</th>
<th>Decide</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Grapefruit</td>
<td># of Nectarines</td>
<td>Total Pieces of Fruit</td>
</tr>
<tr>
<td>Trial 1:</td>
<td>11</td>
<td>33</td>
</tr>
<tr>
<td>Trial 2:</td>
<td>10</td>
<td>30</td>
</tr>
</tbody>
</table>

Start another trial. Our total was 40; the total needed is 36, so our trial started too high and our next trial should start still lower.

<table>
<thead>
<tr>
<th>Define</th>
<th>Do</th>
<th>Decide</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Grapefruit</td>
<td># of Nectarines</td>
<td>Total Pieces of Fruit</td>
</tr>
<tr>
<td>Trial 1:</td>
<td>11</td>
<td>33</td>
</tr>
<tr>
<td>Trial 2:</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>Trial 3:</td>
<td>8</td>
<td>24</td>
</tr>
</tbody>
</table>

Start another trial. Our total was 32; the total needed is 36, so our trial started too low and our next one should be higher than 8 but lower than 10.

<table>
<thead>
<tr>
<th>Define</th>
<th>Do</th>
<th>Decide</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Grapefruit</td>
<td># of Nectarines</td>
<td>Total Pieces of Fruit</td>
</tr>
<tr>
<td>Trial 1:</td>
<td>11</td>
<td>33</td>
</tr>
<tr>
<td>Trial 2:</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>Trial 3:</td>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>Trial 4:</td>
<td>9</td>
<td>27</td>
</tr>
</tbody>
</table>

Step 5:  **Declare:** The answer was found. Answer the question in a sentence. There are 9 grapefruit and 27 nectarines in the box.
Example 2

The perimeter of a rectangle is 120 feet. If the length of the rectangle is ten feet more than the width, what are the dimensions (length and width) of the rectangle?

Describe/Draw:

\[
\begin{array}{c}
\text{width} \\
\text{width + 10}
\end{array}
\]

Start with the width because, of the two required answers, it is the one we know the least about. The length is 10 feet more than the width, so add 10 to the first trial.

<table>
<thead>
<tr>
<th>Define</th>
<th>Do</th>
<th>Decide</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>Length</td>
<td>Perimeter</td>
</tr>
<tr>
<td>Trial 1: 10</td>
<td>20</td>
<td>((10 + 20) \cdot 2 = 60)</td>
</tr>
</tbody>
</table>

Since the trial of 10 resulted in an answer that is too low, we should increase the number in the next trial. Pay close attention to the result of each trial. Each result helps determine the next trial as you narrow down the possible trials to reach the answer. Note: As students get more experience with using the 5-D process, they learn to make better-educated trials from one step to the next to solve problems quickly or to establish the pattern they need to write an equation.

<table>
<thead>
<tr>
<th>Define</th>
<th>Do</th>
<th>Decide</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>Length</td>
<td>Perimeter</td>
</tr>
<tr>
<td>Trial 1: 10</td>
<td>20</td>
<td>((10 + 20) \cdot 2 = 60)</td>
</tr>
<tr>
<td>Trial 2: 20</td>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td>Trial 3: 30</td>
<td>40</td>
<td>140</td>
</tr>
<tr>
<td>Trial 4: 25</td>
<td>35</td>
<td>120</td>
</tr>
</tbody>
</table>

Declare: The dimensions are 25 and 35 feet.
Example 3

Jorge has some dimes and quarters. He has 10 more dimes than quarters and the collection of coins is worth $2.40. How many dimes and quarters does Jorge have?

Note: This type of problem is more difficult than others because the number of things asked for is different than their value. Separate columns for each part of the problem must be added to the table as shown below. Students often neglect to write the third and fourth columns.

Describe: The number of quarters plus 10 equals the number of dimes.
The total value of the coins is $2.40.

<table>
<thead>
<tr>
<th>Define</th>
<th>Do</th>
<th>Decide</th>
</tr>
</thead>
<tbody>
<tr>
<td># Quarters</td>
<td># Dimes</td>
<td>Value of Quarters</td>
</tr>
<tr>
<td>Trial 1:</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Trial 2:</td>
<td>8</td>
<td>18</td>
</tr>
<tr>
<td>Trial 3:</td>
<td>6</td>
<td>16</td>
</tr>
<tr>
<td>Trial 4:</td>
<td>4</td>
<td>14</td>
</tr>
</tbody>
</table>

Declare: Jorge has four quarters and 14 dimes.

HELPFUL QUESTIONS TO ASK YOUR CHILD

When your child is having difficulty with a 5-D problem, it may be because he/she does not understand the problem, not because he/she does not understand the 5-D process. Here are some helpful questions to ask when your child does not understand the problem. (These are useful in non-word problem situations, too.)

1. What are you being asked to find?
2. What information have you been given?
3. Is there any unneeded information? If so, what is it?
4. Is there any necessary information that is missing? If so, what information do you need?
TIPS ABOUT COLUMN TITLES

1. You may select any number for the first trial. Ten of the students’ ages are adequate numbers for the first trial. The result will help you to determine the number to use for the second trial.

2. Continue establishing columns by asking, “What else do we need to know to determine whether the number we used for our trial is correct or too low or too high?”

3. Put the answer to one calculation in each column. Students sometimes try to put the answer to several mental calculations in one column. (See the note in Example 3 on the previous page.)

Problems

Solve these problems using the 5-D process. Write each answer in a sentence.

1. A wood board 100 centimeters long is cut into two pieces. One piece is 26 centimeters longer than the other. What are the lengths of the two pieces?

2. Thu is five years older than her brother Tuan. The sum of their ages is 51. What are their ages?

3. Tomas is thinking of a number. If he triples his number and subtracts 13, the result is 305. What is the number that Tomas is thinking about?

4. Two consecutive numbers have a sum of 123. What are the two numbers?

5. Two consecutive even numbers have a sum of 246. What are the numbers?

6. Joe’s age is three times Aaron’s age and Aaron is six years older than Christina. If the sum of their ages is 149, what is Christina’s age? Joe’s age? Aaron’s age?

7. Farmer Fran has 38 barnyard animals, consisting of only chickens and goats. If these animals have 116 legs, how many of each type of animal are there?

8. A wood board 156 centimeters long is cut into three parts. The two longer parts are the same length and are 15 centimeters longer than the shortest part. How long are the three parts?

9. Juan has 15 coins, all nickels and dimes. This collection of coins is worth 90¢. How many nickels and dimes are there? (Hint: Create separate column titles for, “Number of Nickels,” “Value of Nickels,” “Number of Dimes,” and “Value of Dimes.”)
10. Tickets to the school play are $5.00 for adults and $3.50 for students. If the total value of all the tickets sold was $2517.50 and 100 more students bought tickets than adults, how many adults and students bought tickets?

11. A wood board 250 centimeters long is cut into five pieces: three short ones of equal length and two that are both 15 centimeters longer than the shorter ones. What are the lengths of the boards?

12. Conrad has a collection of three types of coins: nickels, dimes, and quarters. There is an equal amount of nickels and quarters but three times as many dimes. If the entire collection is worth $9.60, how many nickels, dimes, and quarters are there?

**Answers**

1. The lengths of the boards are 37 cm and 63 cm.

2. Thu is 28 years old and her brother is 23 years old.

3. Tomas is thinking of the number 106.

4. The two consecutive numbers are 61 and 62.

5. The two consecutive numbers are 142 and 144.

6. Christine is 25, Aaron is 31, and Joe is 93 years old.

7. Farmer Fran has 20 goats and 18 chickens.

8. The lengths of the boards are 42, 57, and 57 cm.

9. Juan has 12 nickels and 3 dimes.

10. There were 255 adult and 355 student tickets purchased for the play.

11. The lengths of the boards are 44 and 59 cm.

12. Conrad has 16 nickels and quarters and 48 dimes.