DIVISION BY FRACTIONS

6.1.1 – 6.1.4

Division by fractions introduces three methods to help students understand how dividing by fractions works. In general, think of division for a problem like $8 \div 2$ as, “In 8, how many groups of 2 are there?” Similarly, $\frac{1}{2} \div \frac{1}{4}$ means, “In $\frac{1}{2}$, how many fourths are there?”

For more information, see the Math Notes boxes in Lessons 7.2.2 and 7.2.4 of the Core Connections, Course 1 text. For additional examples and practice, see the Core Connections, Course 1 Checkpoint 8B materials. The first two examples show how to divide fractions using a diagram.

Example 1

Use the rectangular model to divide: $\frac{1}{2} \div \frac{1}{4}$.

Step 1: Using the rectangle, we first divide it into 2 equal pieces. Each piece represents $\frac{1}{2}$. Shade $\frac{1}{2}$ of it.

Step 2: Then divide the *original* rectangle into four equal pieces. Each section represents $\frac{1}{4}$. In the shaded section, $\frac{1}{2}$, there are 2 fourths.

Step 3: Write the equation.

$$\frac{1}{2} \div \frac{1}{4} = 2$$

Example 2

In $\frac{3}{4}$, how many $\frac{1}{2}$s are there?

That is, $\frac{3}{4} \div \frac{1}{2} = ?$

Start with $\frac{3}{4}$.

In $\frac{3}{4}$ there is one full $\frac{1}{2}$ shaded and half of another one (that is half of one half).

So: $\frac{3}{4} \div \frac{1}{2} = 1 \frac{1}{2}$

(one and one-half halves)
Problems

Use the rectangular model to divide.

1. \(1 \frac{1}{3} ÷ \frac{1}{6}\)
2. \(\frac{3}{2} ÷ \frac{3}{4}\)
3. \(1 ÷ \frac{1}{4}\)
4. \(1 \frac{1}{4} ÷ \frac{1}{2}\)
5. \(2 \frac{2}{3} ÷ \frac{1}{9}\)

Answers

1. \(8\) thirds
   
   sixths
   
   8 sixths

2. \(2\) halves
   
   fourths
   
   2 three fourths

3. \(4\) one
   
   fourths
   
   4 fourths

4. \(2 \frac{1}{2}\) fourths
   
   halves
   
   2 \(\frac{1}{2}\) halves

5. \(24\) thirds
   
   ninths
   
   24 ninths

The next two examples use common denominators to divide by a fraction. Express both fractions with a common denominator, then divide the first numerator by the second.

Example 3

\[
\frac{4}{3} + \frac{2}{3} =
\]

\[
12 + \frac{10}{15} =
\]

\[
\frac{12}{10} = \frac{6}{5} \text{ or } 1 \frac{1}{5}
\]

Example 4

\[
1 \frac{1}{3} + \frac{1}{6} =
\]

\[
\frac{4}{3} + \frac{1}{6} =
\]

\[
\frac{8}{6} + \frac{1}{6} = \frac{8}{1} \text{ or } 8
\]
One more way to divide fractions is to use the Giant One from previous work with fractions to create a “Super Giant One.” To use a Super Giant One, write the division problem in fraction form, with a fraction in both the numerator and the denominator. Use the reciprocal of the denominator for the numerator and the denominator in the Super Giant One, multiply the fractions as usual, and simplify the resulting fraction when possible.

**Example 5**

\[
\frac{1}{4} \div \frac{4}{4} = \frac{1}{4} \cdot \frac{4}{4} = \frac{4}{2} = 2
\]

**Example 6**

\[
\frac{3}{6} \div \frac{6}{1} = \frac{3}{6} = \frac{18}{2} = 4 \frac{1}{2}
\]

**Example 7**

\[
\frac{1 \frac{1}{2}}{} \cdot \frac{2 \frac{3}{2}}{1} = \frac{8}{2} \cdot \frac{8}{9}
\]

**Example 8**

\[
\frac{2}{3} + \frac{3}{5} \Rightarrow \frac{10}{15} + \frac{9}{15} \Rightarrow \frac{10}{9}
\]

**Problems**

Complete the division problems below. Use any method.

1. \(\frac{3}{7} + \frac{1}{8}\)  
2. \(1 \frac{3}{7} + \frac{1}{2}\)  
3. \(\frac{4}{7} + \frac{1}{3}\)  
4. \(1 \frac{4}{7} + \frac{1}{3}\)  
5. \(\frac{6}{7} + \frac{5}{8}\)

6. \(\frac{3}{10} + \frac{5}{7}\)  
7. \(2 \frac{3}{8} + \frac{5}{8}\)  
8. \(7 \div \frac{1}{3}\)  
9. \(1 \frac{1}{3} + \frac{2}{5}\)  
10. \(2 \frac{2}{3} + \frac{3}{4}\)

11. \(3 \frac{1}{5} + \frac{5}{6}\)  
12. \(1 \frac{1}{2} + \frac{1}{2}\)  
13. \(\frac{5}{8} + \frac{1}{4}\)  
14. \(10 \frac{1}{3} + \frac{1}{6}\)  
15. \(\frac{3}{5} + 6\)

**Answers**

1. \(3 \frac{3}{7}\)  
2. \(2 \frac{6}{7}\)  
3. \(1 \frac{5}{7}\)  
4. \(4 \frac{5}{7}\)  
5. \(1 \frac{13}{35}\)

6. \(\frac{21}{50}\)  
7. \(3 \frac{11}{15}\)  
8. \(21\)  
9. \(3 \frac{1}{3}\)  
10. \(3 \frac{5}{9}\)

11. \(4\)  
12. \(3\)  
13. \(\frac{1}{2}\)  
14. \(62\)  
15. \(\frac{1}{10}\)
When students are first given expressions like $3 + 4 \cdot 2$, some students think the answer is 14 and some think the answer is 11. This is why mathematicians decided on a method to simplify an expression that uses more than one operation so that everyone can agree on the answer.

There is a set of rules to follow that provides a consistent way for everyone to evaluate expressions. These rules, called the Order of Operations, must be followed in order to arrive at a correct answer. As indicated by the name, these rules state the order in which the mathematical operations are to be completed.

For additional information, see the Math Notes box in Lesson 6.2.1 of the *Core Connections, Course 1* text.

The first step is to organize the numerical expression into parts called terms, which are single numbers or products of numbers. A numerical expression is made up of a sum or difference of terms.

Examples of numerical terms are: 4, 3(6), 6(9 – 4), 2 · 3², 3(5 + 2³), and $\frac{16-4}{6}$.

For the problem above, $3 + 4 \cdot 2$, the terms are circled at right.  

Each term is simplified separately, giving $3 + 8$. Then the terms are added: $3 + 8 = 11$. Thus, $3 + 4 \cdot 2 = 11$.

### Example 1

To evaluate an expression:  

$2 \cdot 3^2 + 3(6 − 3) + 10$

• Circle each term in the expression.

• Simplify each term until it is one number by:
  - Simplifying the expressions within the parentheses.
  - Evaluating each exponential part (e.g., $3^2$).
  - Multiplying and dividing from left to right.

• Finally, combine terms by adding or subtracting from left to right.

$2 \cdot 3^2 + 3(3) + 10$

$2 \cdot 9 + 3(3) + 10$

$27 + 10$

$37$
Example 2

a. Circle the terms.

b. Simplify inside the parentheses.

c. Simplify the exponents.

d. Multiply and divide from left to right.

Finally, add and subtract from left to right.

Example 3

a. Circle the terms.

b. Multiply and divide left to right, including exponents.

Add or subtract from left to right.

Problems

Circle the terms, then simplify each expression.

1. $5 \cdot 3 + 4$

2. $10 \div 5 + 3$

3. $2(9 - 4) \cdot 7$

4. $6(7 + 3) + 8 \div 2$

5. $15 \div 3 + 7(8 + 1) - 6$

6. $\frac{9}{3} + 5 \cdot 3^2 - 2(14 - 5)$

7. $\frac{20}{6+4} + 7 \cdot 2 + 2$

8. $\frac{5+30}{7} + 6^2 - 18 \div 9$

9. $2^3 + 8 - 16 \div 8 \cdot 2$

10. $25 - 5^2 + 9 - 3^2$

11. $5(17 - 7) + 4 \cdot 3 - 8$

12. $(5 - 2)^2 + (9 + 1)^2$

13. $4^3 + 9(2) \div 6 + (6 - 1)^2$

14. $\frac{18}{3^2} + \frac{5 \cdot 3}{5}$

15. $3(7 - 2)^2 + 8 \div 4 - 6 \cdot 5$

16. $14 \div 2 + 6 \cdot 8 \div 2 - (9 - 3)^2$

17. $\frac{27}{3} + 18 - 9 \div 3 - (3 + 4)^2$

18. $26 \cdot 2 \div 4 - (6 + 4)^2 + 3(5 - 2)^3$

19. $\left(\frac{42+3}{5}\right)^2 + 3^2 - (5 \cdot 2)^2$
### Answers

1. 19  
2. 5  
3. 70  
4. 64  
5. 62  
6. 30  
7. 9  
8. 39  
9. 12  
10. 0  
11. 54  
12. 109  
13. 44  
14. 5  
15. 47  
16. $-5$  
17. $-25$  
18. $-6$  
19. $-10$
Algebraic expressions can be represented by the perimeters of algebra tiles (rectangles and squares) and combinations of algebra tiles. The dimensions of each tile are shown along its sides and the tile is named by its area as shown on the tile itself in the figures at right. When using the tiles, perimeter is the distance around the exterior of a figure. For additional information, see the Math Notes box in Lesson 6.2.4 of the Core Connections, Course 1 text.

**Example 1**

\[ P = 6x + 4 \text{ units} \]

**Example 2**

\[ P = 6x + 8 \text{ units} \]
Problems

Determine the perimeter of each figure.

1. \[ x^2 \] \[ x \] \[ x \] \[ 4x + 4 \text{ un.} \]

2. \[ x^2 \] \[ x \] \[ 4x + 4 \text{ un.} \]

3. \[ x \] \[ x \] \[ 2x + 8 \text{ un.} \]

4. \[ x^2 \] \[ \text{un.} \] \[ 4x + 6 \text{ un.} \]

5. \[ x^2 \] \[ x \] \[ 4x + 2 \text{ un.} \]

6. \[ x^2 \] \[ x \] \[ 4x + 4 \text{ un.} \]

7. \[ x^2 \] \[ x \] \[ 4 + 4 \text{ un.} \]

8. \[ x \] \[ 2x + 4 \text{ un.} \]

Answers

1. \[ 4x + 4 \text{ un.} \]

2. \[ 4x + 4 \text{ un.} \]

3. \[ 2x + 8 \text{ un.} \]

4. \[ 4x + 6 \text{ un.} \]

5. \[ 4x + 4 \text{ un.} \]

6. \[ 4x + 2 \text{ un.} \]

7. \[ 4x + 4 \text{ un.} \]

8. \[ 2x + 4 \text{ un.} \]
Algebraic expressions can also be simplified by combining (adding or subtracting) terms that have the same variable(s) raised to the same powers, into one term. The skill of combining like terms is necessary for solving equations. For additional information, see the Math Notes box in Lesson 6.2.4 of the *Core Connections, Course 1* text.

**Example 1**

Combine like terms to simplify the expression \(3x + 5x + 7x\).

All these terms have \(x\) as the variable, so they are combined into one term, \(15x\).

**Example 2**

Simplify the expression \(3x + 12 + 7x + 5\).

The terms with \(x\) can be combined. The terms without variables (the constants) can also be combined.

\[
\begin{align*}
3x + 12 + 7x + 5 & \quad \text{Note that in the simplified form the term with the variable is listed} \\
3x + 7x + 12 + 5 & \quad \text{before the constant term.} \\
10x + 17 &
\end{align*}
\]

**Example 3**

Simplify the expression \(5x + 4x^2 + 10 + 2x^2 + 2x - 6 + x - 1\).

\[
\begin{align*}
5x + 4x^2 + 10 + 2x^2 + 2x - 6 + x - 1 & \quad \text{Note that terms with the same variable but} \\
4x^2 + 2x^2 + 5x + 2x + x + 10 - 6 - 1 & \quad \text{with different exponents are not combined and} \\
6x^2 + 8x + 3 & \quad \text{are listed in order of decreasing power of the} \\
& \quad \text{variable, in simplified form, with the constant} \\
& \quad \text{term last.}
\end{align*}
\]
Example 4

The algebra tiles, as shown in the Algebra Tiles and Perimeter section, are used to model how to combine like terms.

The large square represents $x^2$, the rectangle represents $x$, and the small square represents one. We can only combine tiles that are alike: large squares with large squares, rectangles with rectangles, and small squares with small squares. If we want to combine: $2x^2 + 3x + 4$ and $3x^2 + 5x + 7$, visualize the tiles to help combine the like terms:

$2x^2$ (2 large squares) + $3x$ (3 rectangles) + 4 (4 small squares) 
+ $3x^2$ (3 large squares) + $5x$ (5 rectangles) + 7 (7 small squares)

The combination of the two sets of tiles, written algebraically, is: $5x^2 + 8x + 11$.

Example 5

Sometimes it is helpful to take an expression that is written horizontally, circle the terms with their signs, and rewrite like terms in vertical columns before you combine them:

$$(2x^2 - 5x + 6) + (3x^2 + 4x - 9)$$

$$2x^2 - 5x + 6$$
+ $3x^2$ + 4x - 9

This procedure may make it easier to identify the terms as well as the sign of each term.

Problems

Combine the following sets of terms.

1. $(2x^2 + 6x + 10) + (4x^2 + 2x + 3)$
2. $(3x^2 + x + 4) + (x^2 + 4x + 7)$
3. $(8x^2 + 3) + (4x^2 + 5x + 4)$
4. $(4x^2 + 6x + 5) - (3x^2 + 2x + 4)$
5. $(4x^2 - 7x + 3) + (2x^2 - 2x - 5)$
6. $(3x^2 - 7x) - (x^2 + 3x - 9)$
7. $(5x + 6) + (-5x^2 + 6x - 2)$
8. $2x^2 + 3x + x^2 + 4x - 3x^2 + 2$
9. $3c^2 + 4c + 7x - 12 + (-4c^2) + 9 - 6x$
10. $2a^2 + 3a^3 - 4a^2 + 6a + 12 - 4a + 2$

Answers

1. $6x^2 + 8x + 13$
2. $4x^2 + 5x + 11$
3. $12x^2 + 5x + 7$
4. $x^2 + 4x + 1$
5. $6x^2 - 9x - 2$
6. $2x^2 - 10x + 9$
7. $-5x^2 + 11x + 4$
8. $7x + 2$
9. $-c^2 + 4c + x - 3$
10. $3a^3 - 2a^2 + 2a + 14$