In addition and multiplication, the order of the numbers can be reversed: $2 + 5 = 5 + 2$ and $2 \cdot 5 = 5 \cdot 2$. This is called the Commutative Property. In symbols:

The Commutative Property of Addition states: $a + b = b + a$ and

The Commutative Property of Multiplication states: $a \cdot b = b \cdot a$.

When adding three numbers or multiplying three numbers, the grouping can be changed: $(2 + 3) + 5 = 2 + (3 + 5)$ and $(2 \cdot 3) \cdot 5 = 2 \cdot (3 \cdot 5)$. This is the Associative Property. In symbols:

The Associative Property of Addition states: $(a + b) + c = a + (b + c)$ and

The Associative Property of Multiplication states: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

The Distributive Property distributes one operation over another. So far in these courses, students have seen multiplication distributed over addition. In symbols:

For all numbers $a$, $b$, and $c$, $a(b + c) = a \cdot b + a \cdot c$.

For example, $2(3 + 5) = 2 \cdot 3 + 2 \cdot 5$.

For additional information see the Math Notes box in Lesson 4.1.1 of the Core Connections, Course 2 text.

The properties of multiplication and addition allow calculations to be rearranged. Doing this is helpful when doing calculations mentally. Name the property or reason that justifies each step.

**Example 1**

Calculate mentally: $4 \cdot (17 \cdot 25)$

<table>
<thead>
<tr>
<th>Step</th>
<th>Calculation</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>$4 \cdot (25 \cdot 17)$</td>
<td>Commutative Property of Multiplication</td>
</tr>
<tr>
<td>Step 2</td>
<td>$(4 \cdot 25) \cdot 17$</td>
<td>Associative Property of Multiplication</td>
</tr>
<tr>
<td>Step 3</td>
<td>$(100) \cdot 17$</td>
<td>mental math</td>
</tr>
<tr>
<td>Step 4</td>
<td>$1700$</td>
<td>mental math</td>
</tr>
</tbody>
</table>

**Example 2**

Calculate mentally: $8(56)$

<table>
<thead>
<tr>
<th>Step</th>
<th>Calculation</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>$8(50 + 6)$</td>
<td>by renaming 56 as $50 + 6$</td>
</tr>
<tr>
<td>Step 2</td>
<td>$8(50) + 8(6)$</td>
<td>Distributive Property</td>
</tr>
<tr>
<td>Step 3</td>
<td>$400 + 48$</td>
<td>mental math</td>
</tr>
<tr>
<td>Step 4</td>
<td>$448$</td>
<td>mental math</td>
</tr>
</tbody>
</table>
Problems

Listed below are possible steps used to mentally calculate a problem. Give the missing reasons that justify the steps.

1. \[ 15(29) = 15(30 + (-1)) \]  
   \[ 15(30 - 1) = 15(30) + 15(-1) \]  
   \[ 150 + (-15) = 150 + (-10 + -5) \]  
   \[ 150 + (-10) + (-5) = (150 + (-10)) + (-5) \]  
   \[ 140 + (-5) = 135 \]  
   renamed 29 as 30 + (-1)  
   a. Distributive  
   b. Associative  
   mental math

2. \[ 386 + 177 + 214 = 386 + 214 + 177 \]  
   \[ 386 + 214 + 177 = (386 + 200) + 14 + 177 \]  
   \[ 586 + 14 + 177 = (586 + 14) + 177 \]  
   \[ 600 + 177 = 777 \]  
   a. Commutative  
   b. Associative  
   c. Associative  
   mental math

3. \[ 49(12) = 12(49) \]  
   \[ 12(49) = 12(50 - 1) \]  
   \[ 12(50 - 1) = 12(50) - 12(1) \]  
   \[ 12(50) - 12 = (6 \cdot 2)(50) - 12 \]  
   \[ (6 \cdot 2)(50) - 12 = 6(2 \cdot 50) - 12 \]  
   \[ 600 - 12 = 588 \]  
   renamed 49 as 50 – 1  
   a. Commutative  
   b. Distributive  
   c. Associative  
   mental math

Answers

1. a. Distributive  
   b. Associative

2. a. Commutative  
   b. Associative  
   c. Associative

3. a. Commutative  
   b. Distributive  
   c. Associative