PROPORTIONAL RELATIONSHIPS 4.2.1, 4.2.2, and 4.2.4

A proportion is an equation stating the two ratios (fractions) are equal. Two values are in a proportional relationship if a proportion may be set up to relate the values.

For more information, see the Math Notes boxes in Lessons 4.2.3, 4.2.4, and 7.2.2 of the Core Connections, Course 2 text. For additional examples and practice, see the Core Connections, Course 2 Checkpoint 9 materials.

Example 1

The average cost of a pair of designer jeans has increased $15 in 4 years. What is the unit growth rate (dollars per year)?

Solution: The growth rate is $\frac{15 \text{ dollars}}{4 \text{ years}}$. To create a unit rate we need a denominator of “one.”

\[
\frac{15 \text{ dollars}}{4 \text{ years}} = \frac{x \text{ dollars}}{1 \text{ year}}.
\]

Using a Giant One:

\[
\frac{15 \text{ dollars}}{4 \text{ years}} \cdot \frac{1 \text{ year}}{x \text{ dollars}} \Rightarrow \frac{15}{4} = \frac{x}{1} \Rightarrow x = 3.75 \text{ dollars per year}.
\]

Example 2

Ryan’s famous chili recipe uses 3 tablespoons of chili powder for 5 servings. How many tablespoons are needed for the family reunion needing 40 servings?

Solution: The rate is $\frac{3 \text{ tablespoons}}{5 \text{ servings}}$ so the problem may be written as a proportion: $\frac{3}{5} = \frac{t}{40}$.

One method of solving the proportion is to use the Giant One:

\[
\frac{3}{5} = \frac{t}{40} \Rightarrow \frac{3}{5} \cdot \frac{40}{5} = \frac{t}{40} \Rightarrow t = 24
\]

Another method is to use cross multiplication:

\[
\frac{3}{5} = \frac{t}{40} \Rightarrow \frac{3 \cdot 40}{5} = \frac{t \cdot 5}{5} \Rightarrow 3 \cdot 40 = 5t \Rightarrow 5t = 120 \Rightarrow t = 24.
\]

Finally, since the unit rate is $\frac{3}{5}$ tablespoon per serving, the equation $t = \frac{3}{5} \cdot 40$ represents the general proportional situation and one could substitute the number of servings needed into the equation: $t = \frac{3}{5} \cdot 40 = 24$. Using any method the answer is 24 tablespoons.
Example 3

Based on the table at right, what is the unit growth rate (meters per year)?

Solution: \[
\frac{2 \text{ meters}}{10 \text{ years}} = \frac{2 \text{ meters} \times \frac{1}{10}}{10 \text{ years} \times \frac{1}{10}} = \frac{1}{5} \text{ meter per year}
\]

<table>
<thead>
<tr>
<th>height (m)</th>
<th>15</th>
<th>17</th>
</tr>
</thead>
<tbody>
<tr>
<td>years</td>
<td>75</td>
<td>85</td>
</tr>
</tbody>
</table>

Problems

For problems 1 through 10 find the unit rate. For problems 11 through 25, solve each problem.

1. Typing 731 words in 17 minutes (words per minute)
2. Reading 258 pages in 86 minutes (pages per minute)
3. Buying 15 boxes of cereal for $43.35 (cost per box)
4. Scoring 98 points in a 40 minute game (points per minute)
5. Buying 2 \(\frac{1}{4}\) pounds of bananas cost $1.89 (cost per pound)
6. Buying 2 \(\frac{1}{3}\) pounds of peanuts for $2.25 (cost per pound)
7. Mowing 1 \(\frac{1}{2}\) acres of lawn in \(\frac{3}{4}\) of a hour (acres per hour)
8. Paying $3.89 for 1.7 pounds of chicken (cost per pound)
9. \[
\begin{array}{c|c|c|c|c|c}
\text{weight (g)} & 6 & 8 & 12 & 20 \\
\text{length (cm)} & 15 & 20 & 30 & 50 \\
\end{array}
\]
What is the weight per cm?
10. For the graph at right, what is the rate in miles per hour?
11. If a box of 100 pencils costs $4.75, what should you expect to pay for 225 pencils?
12. When Amber does her math homework, she finishes 10 problems every 7 minutes. How long will it take for her to complete 35 problems?
13. Ben and his friends are having a TV marathon, and after 4 hours they have watched 5 episodes of the show. About how long will it take to complete the season, which has 24 episodes?
14. The tax on a $600 vase is $54. What should be the tax on a $1700 vase?
15. Use the table at right to determine how long it will take the Spirit club to wax 60 cars.

<table>
<thead>
<tr>
<th>cars waxed</th>
<th>8</th>
<th>16</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>hours</td>
<td>3</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

16. While baking, Evan discovered a recipe that required \( \frac{1}{2} \) cups of walnuts for every \( 2 \frac{1}{4} \) cups of flour. How many cups of walnuts will he need for 4 cups of flour?

17. Based on the graph, what would the cost to refill 50 bottles?

18. Sam grew \( 1 \frac{3}{4} \) inches in \( 4 \frac{1}{2} \) months. How much should he grow in one year?

19. On his afternoon jog, Chris took 42 minutes to run \( 3 \frac{3}{4} \) miles. How many miles can he run in 60 minutes?

20. If Caitlin needs \( 1 \frac{1}{3} \) cans of paint for each room in her house, how many cans of paint will she need to paint the 7-room house?

21. Stephen receives 20 minutes of video game time every 45 minutes of dog walking he does. If he wants 90 minutes of game time, how many hours will he need to work?

22. Sarah’s grape vine grew 15 inches in 6 weeks, write an equation to represent its growth after \( t \) weeks.

23. On average Max makes 45 out of 60 shots with the basketball, write an equation to represent the average number of shots made out of \( x \) attempts.

24. Write an equation to represent the situation in problem 14.

25. Write an equation to represent the situation in problem 17.

**Answers**

1. 43 words/minute
2. 3 pages/minute
3. 2.89 \( \frac{\text{dollars}}{\text{box}} \)
4. 2.45 \( \frac{\text{points}}{\text{minute}} \)
5. 0.84 \( \frac{\text{dollars}}{\text{pound}} \)
6. 3.38 \( \frac{\text{dollars}}{\text{pound}} \)
7. 2 \( \frac{\text{acres}}{\text{hour}} \)
8. 2.29 \( \frac{\text{dollars}}{\text{pound}} \)
9. \( \frac{2}{5} \) grams/centimeter
10. \( \approx 27 \) miles/hour
11. $10.69
12. 24.5 min.
13. 19.2 hours
14. $153
15. 22.5 hours
16. \( \frac{8}{9} \) cup
17. $175
18. 4 \( \frac{2}{3} \) inches
19. \( \approx 5.36 \) miles
20. 9 \( \frac{1}{3} \) cans
21. 3 \( \frac{3}{8} \) hours
22. \( g = \frac{5}{2} t \)
23. \( s = \frac{3}{4} x \)
24. \( t = 0.09c \)
25. \( C = 3.5b \)

© 2013 CPM Educational Program. All rights reserved. Core Connections, Course 2