**Intercepts and Intersections** both represent a point at which two paths cross, but an intercept specifies where a curve or line crosses an axis, whereas an intersection refers to any point where the graphs of two equations cross. When the graph is given, the points may be estimated from the graph. For more accuracy, intercepts and intersections may be found using algebra in most cases.

**Example 1**

Use the graphs of the parabola $y = x^2 - 3x - 10$ and the line $y = -2x + 2$ at right. The graph shows that the parabola has two $x$-intercepts $(-2, 0)$ and $(5, 0)$ and one $y$-intercept $(0, -10)$. The line has $x$-intercept $(1, 0)$ and $y$-intercept $(0, 2)$. The parabola and the line cross each other twice yielding two points of intersection: $(-3, 8)$ and $(4, -6)$.

Recall that $x$-intercepts are always of the form $(x, 0)$ so they can be found by making $y = 0$ and solving for $x$. Similarly, $y$-intercepts are always of the form $(0, y)$ so they can be found by making $x = 0$ and solving for $y$. To find the points of intersection, solve the system of equations. See the algebraic solutions below.

**Intercepts**

To find the $x$-intercepts of the parabola, make $y = 0$ and solve for $x$.

$$0 = x^2 - 3x - 10$$

Factor and use the Zero Product Property.

$$0 = (x - 5)(x + 2)$$

$x = 5$ or $x = -2$,
so $(5, 0)$ and $(-2, 0)$ are the $x$-intercepts.

To find the $y$-intercepts of the parabola, make $x = 0$ and solve for $y$.

$$y = 0^2 - 3 \cdot 0 - 10 = -10$$

so $(0, -10)$ is the $y$-intercept.

To find the $x$- and $y$-intercepts of the line follow the same procedure.

If $x = 0$, then $y = -2 \cdot 0 + 2 = 2$ so $(0, 2)$ is the $y$-intercept.

If $y = 0$, then $0 = -2x + 2$, and $x = 1$ so $(1, 0)$ is the $x$-intercept.

**Intersection**

To find the point(s) of intersection of the system of equations use the Equal Values Method or substitution.

$$x^2 - 3x - 10 = -2x + 2$$

Make one side equal to zero, factor, and use the Zero Product Property to solve for $x$.

(The Quadratic Formula is also possible.)

$$x^2 - x - 12 = 0$$

$(x - 4)(x + 3) = 0$

$x = 4$ or $x = -3$

Substituting $x = 4$ into either equation yields $y = -6$, so $(4, -6)$ is a point of intersection.

Substituting $x = -3$ into either equation yields $y = 8$, so $(-3, 8)$ is a point of intersection.
Example 2

Given the lines \( y = \frac{1}{3} x + 8 \) and \( y = \frac{1}{2} x - 3 \), determine their intercepts and intersection without graphing. Using the same methods as shown in Example 1:

### Intercepts

For the line \( y = \frac{1}{3} x + 8 \):

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>( \frac{1}{3} \cdot 0 + 8 )</td>
<td>8</td>
</tr>
</tbody>
</table>

If \( x = 0 \), then \( y = \frac{1}{3} \cdot 0 + 8 = 8 \), so \((0, 8)\) is the \( y \)-intercept.

If \( y = 0 \), then \( \frac{1}{3} x + 8 = 0 \), and \( x = -24 \), so \((-24, 0)\) is the \( x \)-intercept.

For the line \( y = \frac{1}{2} x - 3 \):

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>( \frac{1}{2} \cdot 0 - 3 )</td>
<td>-3</td>
</tr>
</tbody>
</table>

If \( x = 0 \), then \( y = \frac{1}{2} \cdot 0 - 3 = -3 \), so \((0, -3)\) is the \( y \)-intercept.

If \( y = 0 \), then \( \frac{1}{2} x - 3 = 0 \), and \( x = 6 \), so \((6, 0)\) is the \( x \)-intercept.

### Intersection

\( \frac{1}{3} x + 8 = \frac{1}{2} x - 3 \)

\( 6 \cdot \left( \frac{1}{3} x + 8 = \frac{1}{2} x - 3 \right) \)

\( 2x + 48 = 3x - 18 \)

\( 66 = x \)

Substituting \( x = 66 \) into either original equation yields \( y = 30 \).

The point of intersection is \((66, 30)\).

### Problems

For each equation, determine the \( x \)- and \( y \)-intercepts.

1. \( y = 3x - 2 \)
2. \( y = \frac{1}{4} x - 2 \)
3. \( 3x + 2y = 12 \)
4. \( -x + 3y = 15 \)
5. \( 2y = 15 - 3x \)
6. \( y = x^2 + 5x + 6 \)
7. \( y = x^2 - 3x - 10 \)
8. \( y = 4x^2 - 11x - 3 \)
9. \( y = x^2 + 3x - 2 \)

Determine the point(s) of intersection

10. \( y = 5x + 1 \)
    \( y = -3x - 15 \)
11. \( y = x + 7 \)
    \( y = 4x - 5 \)
12. \( x = 7 + 3y \)
    \( x = 4y + 5 \)
13. \( x = -\frac{1}{2}y + 4 \)
    \( 8x + 3y = 31 \)
14. \( 4x - 3y = -10 \)
    \( x = \frac{1}{4} y - 1 \)
15. \( 2x - y = 6 \)
    \( 4x - y = 12 \)
16. \( y = x^2 - 3x - 8 \)
    \( y = 2 \)
17. \( y = x^2 - 7 \)
    \( y = 8 + 2x \)
18. \( y = x^2 + 2x + 8 \)
    \( y = -4x - 1 \)