The simplest equation of one shape (e.g., line, parabola, absolute value) is called a parent equation. Changing a parent equation by addition or multiplication moves and changes the size and orientation of the parent graph but does not change the basic shape. These changes are called transformations. The first set of examples shows how the parent graph of a parabola can be moved on an $xy$-coordinate system. In later courses, you will learn how to make the parabola wider or narrower. Transformations of other functions are done in a similar manner.

**Examples**

- $f(x) = x^2$  
  - the parent graph
- $f(x - 3) = (x - 3)^2$  
  - right 3 units
- $f(x + 2) = (x + 2)^2$  
  - left 2 units
- $f(x) - 2 = x^2 - 2$  
  - down 2 units
- $f(x + 3) - 2 = (x + 3)^2 - 2$  
  - left 3 and down 2 units
- $-f(x - 2) + 5 = -(x - 2)^2 + 5$  
  - right 2 and up 5 units, and reflected across $x$-axis.
Problems

Predict how each parabola is different from the parent graph. For each of these problems, the parent graph is $f(x) = x^2$.

1. $y = f(x + 5)$
2. $y = f(x) + 5$
3. $y = -f(x)$
4. $y = f(x) - 4$
5. $y = f(x - 5)$
6. $y = -f(x + 2)$
7. $y = f(x - 5) - 3$
8. $y = -f(x + 2) + 1$
9. $y = f(x - 3) - 5$

The parent graphs for absolute value and square root are shown at right. They are transformed exactly the same way as parabolas. Predict how the graph of each equation below is different from the parent graph.

10. $f(x - 4)$ where $f(x) = |x|
11. f(x) + 5$ where $f(x) = \sqrt{x}$
12. $f(x) + 3$ where $f(x) = |x|
13. f(x + 2)$ where $f(x) = \sqrt{x}
14. -f(x) + 5$ where $f(x) = |x|
15. -f(x)$ where $f(x) = \sqrt{x}
16. -f(x + 3)$ where $f(x) = \sqrt{x}
17. -f(x - 5)$ where $f(x) = |x|
18. -f(x - 2) + 5$ where $f(x) = \sqrt{x}$
19. $f(x) + 3$ where $f(x) = \sqrt{x}
20. f(x - 7)$ where $f(x) = \sqrt{x}$
21. -f(x + 3) + 7 where $f(x) = |x|

Answers

1. left 5
2. up 5
3. reflected across x-axis
4. down 4
5. left 5
6. left 2, reflected across x-axis
7. right 5, down 3
8. left 2, up 1, reflected across x-axis
9. right 3, down 5
10. right 4
11. up 5
12. up 3
13. left 2
14. up 5, reflected across x-axis
15. reflected across x-axis
16. left 3, reflected across x-axis
17. right 5, reflected across x-axis
18. right 2, up 5, reflected across x-axis
19. up 3
20. right 7
21. left 3, up 7, reflected across x-axis
DATA DISTRIBUTIONS can be represented graphically with histograms and boxplots. Boxplots are described in the Math Notes box in Lesson 11.2.1. For assistance with histograms, see the Parent Guide with Extra Practice for Lesson 1.1.4 of Core Connections, Course 1, which is available free of charge at cpm.org.

Two distributions of data can be compared by comparing their center, shape, spread, and outliers.

The center, or “typical” value, of a data distribution can be described by the median. If the distribution is symmetric and has no outliers, the mean can be used to describe the center.

Common shapes of data distributions can be found in the Math Notes box in Lesson 11.2.2.

The spread of a distribution can be described with the interquartile range (IQR), described in the Math Notes boxes in Lesson 11.2.1, or the standard deviation, described in the Math Notes box in Lesson 11.2.3. Since the standard deviation is based upon the mean, it should be used only to describe the spread of distributions that are symmetric and without outliers.

An outlier is a data value that is far away from the bulk of the data.

Example 1

University professors are complaining that the English Literature classes at community colleges are not demanding enough. Specifically, the university professors claim that community college literature courses are not assigning enough novels to read. A community college statistics student collected the following data from 42 universities and community college literature courses in the state. Compare the number of novels read in the two types of colleges.

Number of novels assigned in community college literature courses:
13, 10, 15, 12, 14, 9, 11, 15, 12, 14, 9, 10, 13, 15, 12, 9, 11, 15, 12, 10, 15, 14  checksum 270

Number of novels assigned in university literature courses:
11, 8, 14, 13, 25, 11, 7, 13, 8, 16, 11, 10, 20, 7, 8, 13, 14, 16, 18, 10  checksum 253

Solution follows on next page →
Solution

Any analysis of data distributions should begin with a graphical representation of the data. A bin width of two was chosen for the histograms that follow. So that the distributions can be compared, both graphs have the same scale on the $x$-axis, and the graphs are stacked on top of each other. For assistance with using a TI-83+/84+ calculator, refer to your eBook or visit studenhelp.cpm.org.

The checksum is used to verify that data has been entered into the graphing technology correctly. The sum of the data set, as determined by the statistical functions of the calculator, should match the given checksum value.

When comparing the distributions, the center, shape, spread, and outliers should be considered. Since neither of the distributions is nearly symmetric, and one of the distributions has an outlier, it would not be appropriate to use the means or standard deviations to compare. The five number summaries (see the Math Notes box in Lesson 11.2.1) are shown to the right of each graph.

**Center:** Both types of colleges assign the same median of 12 novels.

*Solution continues on next page →*
Solution continued from previous page.

**Shape:** The distribution for community colleges is skewed, with a low of 8 to 9 novels and increasing to a peak at 14 to 15 novels. The distribution at universities is skewed in the other direction, with a peak at 10 to 11 novels.

**Spread:** The variability in the number of novels assigned at the community college level is much less than the variability between courses at the university level. The IQR for community colleges is 4 novels (14 – 10 = 4), while the university IQR of 6 (15 – 9 = 6) is one-and-a-half times as wide.

**Outliers:** One course at a university is an outlier; 25 books are assigned in that course. Twenty-five books is far away from the bulk of the university courses. The TI-83/84+ calculator can mark an outlier on a boxplot with dots.

**Conclusions:** The university professors claim that their courses are more demanding because they assign more novels. However, that data does not bear this claim out. 25% of university courses assign more novels than any of the community college courses (the right “whisker,” or the top 25% of the courses, for universities is beyond the entire boxplot for community colleges). But just as dramatically, 25% of the university classes assign fewer books than any of the community colleges (the left “whisker,” or lowest 25%, for universities is below the entire boxplot for community colleges). Furthermore, the median number of novels assigned at the two universities is the same—12 books. Community colleges are more consistent from course-to-course in the number of novels they assign (IQR is 4) than are the universities (IQR is 6).

**Example 2**

A rabbit breeder kept track of the number of offspring from five does (female rabbits) this year. The does had: 243, 215, 184, 280, and 148 kits (baby rabbits) respectively. Show how to calculate the mean and standard deviation of the number of kits per doe without using the statistical functions of a calculator.

The mean is \[
\frac{243 + 215 + 184 + 280 + 148}{5} = 214 \text{ kits}.
\]

The standard deviation is the square root of the average of the distances to the mean, after the distances have been made positive by squaring. To find the standard deviation, first find the distance each doe is from the mean:

- \[243 - 214 = 29,\]
- \[215 - 214 = 1,\]
- \[184 - 214 = -30,\]
- \[280 - 214 = 66,\]
- \[148 - 214 = -66.\]

Find each of the distances squared: \[29^2 = 841,\]
\[1^2 = 1,\]
\[(-30)^2 = 900,\]
\[66^2 = 4356,\]
\[(-66)^2 = 4356.\]

The mean distance-squared is: \[
\frac{841 + 1 + 900 + 4356 + 4356}{5} = 2090.8
\]

The square root is 45.725. Since the precision of the original measurements was an integer, the final result should also be an integer. The mean number of kits per doe is 214 with a standard deviation of 46 kits.
Problems

1. Different types of toads tend to lay different numbers of eggs. The following data was collected from two different species. Compare the number of eggs laid by American toads to the number laid by Fowler toads. Is it appropriate to summarize the distributions by using mean and standard deviation? Use a bin width of 250 eggs.

American toads: 9100, 8700, 10300, 9500, 7800, 8900, 9200, 9300, 8900, 8300, 9400, 8000, 9000, 8400, 9700, 10000, 8600, 8900, 9900, 9300 checksum 181,200.

Fowler Toads: 9500, 9100, 9400, 8800, 9000, 8400, 9200, 9200, 8900, 9100, 8600, 9200, 8700, 9800, 9300, 8800, 9200, 9300, 9000, 9100 checksum 181,600.

2. Without using the statistical functions on your calculator, find the standard deviation of the number of eggs laid by each of the first five American toads.

3. Do low birth weight babies start crawling at a later age than babies born at an average weight? A psychologist collected the following data for the age at which children started crawling:

low weight babies: 10, 12, 11, 11, 7, 13, 10, 12, 11, 13, 10, 11, 15, 11, 14, 10 months checksum 181.

average weight babies: 7, 6, 13, 9, 8, 7, 5, 7, 9, 8, 10, 8, 11, 7, 7, 10, 6, 8, 7, 6, 12, 8, 7 months checksum 186.

4. Compare the amount of time the flavor lasted for people chewing brand “10” chewing gum to the amount of time the flavor lasted in “Strident” chewing gum. See the graph at right. Estimate the mean for each type of gum.
**Answers**

1. Mean and standard deviation are appropriate statistics because both distributions are fairly symmetric with no outliers.

Both types of toads lay a mean of between 9000 and 9100 eggs. Both distributions are single-peaked and symmetric with no apparent outliers. However there is much greater variability in the number of eggs that American toads lay. The standard deviation for American toads is about 637 while the standard deviation for Fowler toads is only half as much, about 316 eggs.

2. \[ \sqrt{\frac{20^2 + (-380)^2 + 1220^2 + 420^2 + (-1280)^2}{5}} \approx 830 \text{ eggs} \]
3. The median and IQR will be used to compare statistics since mean and standard deviation are not appropriate—both distributions are skewed and one has an outlier.

The median age at which low-weight babies start crawling is 11 months, while the median age for average-weight babies is 8 months.

Both distributions are single-peaked and skewed. The low-weight babies appear to have an outlier at 7 months, although the calculator does not identify it as a true outlier. The average-weight babies have an outlier at 13 months.

The variability in the crawling age is roughly the same for low-weight babies (IQR is 2.5 months) as for average-weight babies (IQR is 2 months).

Low-weight babies have their development delayed by about 3 months. About 75% of low-weight babies have not started crawling at the age when 75% of average-weight babies are already crawling.

4. The median for both types of gum was about 18 minutes of flavor time. The times for “10” were skewed, while the times for Strident were symmetric. The lower half of the distributions for both gums was the same. But there was much more variability in the upper half of people chewing “10” than in the upper half of Strident. Indeed, more than 25% of “10” chewers reported flavor lasting longer than any of the Strident chewers. Neither gum had outliers in flavor time.

There was more variability in flavor time for “10”—the IQR was about 9 minutes (25 – 16 = 9). The IQR of 4 minutes (20 – 16 = 4) for Strident was less than half that of “10”. That variability is an advantage. If you chew “10,” you will probably be no worse off than chewing Strident, and you could have much longer flavor.

The mean for Strident is about the same as the median since the distribution is symmetric—about 18 minutes. But the mean for “10” is longer than 18 minutes due to the skew in the shape—maybe 22 minutes or so.