When theoretical probability calculations become too complex, statisticians often use simulations instead. Simulations can also be used to check statistical computations, or they can be used in place of a study which is too expensive, time-consuming, or unethical. Simulations require random numbers or outcomes. In this chapter the simulations are simple enough that students can use coins, dice, tables of random digits, or their calculators to generate required random numbers or outcomes.

Example 1

Dana is looking for clear wood (without knots) to make furniture. The chair he has designed requires 4 three-foot boards of clear wood. If there is a 20% chance of a knot being in a particular foot of wood, on average, how many boards will Dana have to sort through to get enough clear wood for one chair?

Solution: To simulate each 3-foot board, you could use a calculator or random number table to produce random digits in groups of three from 0 to 9. Consider each digit to be a foot of wood within the “board” with zeros and ones to represent knots. Simulated boards would look like \{603\}, \{995\}, \{147\}, etc. with each “foot” having a 20% chance of having a “knot.”

Continue generating your simulated boards counting the number created in order to get 4 sets without a 0 or 1.

Repeat this process enough times to get a meaningful average.

8 boards, 7 boards, 4 boards,
9 boards, 10 boards, 11 boards

*The theoretical mean is approximately 7.81.*
Example 2

Would it be easier for Dana to find a single 12-foot board of clear wood rather than many 3-foot pieces? Use a simulation to estimate the average number of 12-foot boards he would have to consider.

Solution: The mean will be about fifteen boards (theoretical mean approximately 14.55). This is considerably more than the mean boards examined when four 3-foot boards were used, so finding four 3-foot clear boards would be easier.

Problems

1. Jack is planning flights to Fort Lauderdale to catch a cruise ship. The least expensive route will require three separate flights to arrive in Miami and then a shuttle bus connection to Fort Lauderdale. Jack believes that if more than one of the legs is delayed he will miss his ship. Jack researches and finds the airline and bus company both have an 80% on-time rating. If Jack buys the least expensive ticket, what is the probability that Jack misses his ship?

2. Jack looks for an alternative and finds a more expensive flight to Fort Lauderdale with another airline, eliminating the need for the shuttle bus. To afford this ticket he will need to avoid airport parking fees and have his friend Clara drive him to the airport. This airline has an 85% on-time rating but he estimates Clara’s on-time percentage at around 65%.
   a. Use simulation to estimate the probability that more than one leg of this journey is delayed.
   b. Which travel plan would you recommend to Jack, the one with the shuttle bus or the one involving Clara? Why?
Answers

1. To simulate each trip you could use a calculator or random number table to produce random digits in groups of four from 0 to 9. \( \text{randInt}(0, 9, 4) \) Consider each digit to be a leg of his journey with zeros and ones to represent delays. Simulated journeys would look like \( \{5 8 1 1\} \), \( \{4 9 6 0\} \), \( \{7 6 7 3\} \), etc. with each “leg” having a 20% chance of a “delay”. Continue generating simulated trips (about 30 to 50) until you feel you have an accurate proportion of missed cruise ships out of total trip attempts.

```
| 5 7 5 4 | ok  | 6 7 2 1 | ok  | 6 9 9 2 | ok  | 7 9 8 3 | ok  |
| 0 8 0 3 | miss| 8 0 7 9 | ok  | 5 2 7 8 | ok  | 6 5 8 6 | ok  |
| 8 5 1 6 | ok  | 3 0 8 9 | ok  | 5 9 1 1 | miss| 9 5 4 6 | ok  |
| 6 8 8 4 | ok  | 6 4 5 7 | ok  | 1 8 0 6 | miss| 9 0 4 6 | ok  |
| 8 4 9 3 | ok  | 6 5 8 4 | ok  | 1 1 0 8 | miss| 6 1 6 5 | ok  |
```

In this example, 5 missed cruise ships out of 28 trips is \( \frac{5}{28} \approx 0.179 \). The theoretical probability is approximately 0.181.

2. a. To simulate each trip you could use a calculator or random number table to produce random numbers in groups of 3 from 0 to 99. \( \text{randInt}(0, 99, 3) \) Consider each number to be a leg of Jack’s journey with 0 to 34 to represent delays in the first leg with Clara and 0 to 14 to represent delays in the flights. Simulated journeys would now look like \( \{75 8 41\} \), \( \{49 60 76\} \), \( \{72 37 4\} \), etc. Continue generating simulated trips (about 30 to 50) until you feel you have an accurate proportion of missed cruise ships out of total trip attempts.

```
| 46 41 87 | ok  | 66 56 69 | ok  | 69 13 66 | ok  | 16 97 46 | ok  |
| 20 13 17 | ok  | 37 63 61 | ok  | 92 4 62 | ok  | 66 22 59 | ok  |
| 9 41 37 | ok  | 12 81 75 | ok  | 37 62 51 | ok  | 51 93 172 | ok  |
| 7 65 77 | ok  | 63 56 42 | ok  | 20 17 183 | ok  | 13 26 35 | ok  |
| 62 36 35 | ok  | 18 53 98 | miss| 97 7 43 | miss| 98 41 57 | ok  |
| 62 85 25 | ok  | 45 79 61 | ok  | 24 0 123 | ok  | 23 40 78 | ok  |
```

In this example, 3 missed ships out of 28 trips is \( \frac{3}{28} \approx 0.107 \). The theoretical probability is approximately 0.112.

b. Option 2 looks better with a lower chance of missing the ship. The advantages of eliminating one leg of the trip and choosing a more reliable airline make up for Clara’s lower on-time rating.