LOGARITHMS

The earlier sections of this chapter gave students many opportunities to find the inverses of various functions. Here, students explore the inverse of an exponential function. Although they can graph the inverse by reflecting the graph of an exponential function across the line $y = x$, they cannot write the equation of this new function. Writing the equation requires the introduction of a new function, the logarithm. Students explore the properties and graphs of logarithms, and in a later chapter use them to solve equations of this type. For further information see the Math Notes box in Lesson 5.2.2.

Example 1

Find each of the values below and then justify your answer by writing the equivalent exponential form.

a. $\log_5(25) =$ ?

b. $\log_7(?) = 3$

c. $\log_2\left(\frac{1}{8}\right) =$ ?

A logarithm is really just an exponent, so an expression like the one in part (a), $\log_5(25)$, is asking “What exponent can I raise the base 5 to, to get 25?” We can translate this question into an equation: $5^? = 25$. By phrasing it this way, the answer is more apparent: 2. This is true because $5^2 = 25$.

Part (b) can be rephrased as $7^3 = ?$. The answer is 343.

Part (c) asks “2 to what exponent gives $\frac{1}{8}$?” or $2^? = \frac{1}{8}$. The answer is –3 because $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$.

Example 2

The graph of $y = \log(x)$ is shown at right. Use this “parent graph” to graph each of the following equations. Explain how you get your new graphs.

$$y = \log(x - 4) \quad y = 6 \log(x) + 3 \quad y = -\log(x)$$
The logarithm function follows the same rules for transforming its graphs as other functions we have used. The first equation shifts the original graph to the right four units. The graph of the second equation is shifted up three units (because of the “+ 3”) but is also stretched because it is multiplied by six. The third function is flipped across the x-axis. All three of these graphs are shown at right. The original function $y = \log(x)$ is also there, in light gray. Note: When a logarithm is written without a base, as in $y = \log(x)$ and the log key used on a calculator, the base is 10.

Problems

Rewrite each logarithmic equation as an exponential equation and vice versa.

1. $y = \log_4(x)$
2. $3 = \log_2(x)$
3. $x = \log_5(30)$
4. $4^x = 80$
5. $\left(\frac{1}{2}\right)^x = 64$
6. $x^3 = 343$
7. $5^x = \frac{1}{125}$
8. $\log_x(32) = y$
9. $11^3 = x$
10. $-4 = \log_5\left(\frac{1}{16}\right)$

What is the value of $x$ in each equation below? If necessary, rewrite the expression in the equivalent exponential equation to verify your answer.

11. $4 = \log_5(x)$
12. $2 = \log_9(x)$
13. $9 = \log(x)$
14. $81 = 9^x$
15. $\left(\frac{1}{3}\right)^x = 243$
16. $6^x = 7776$
17. $7^x = \frac{1}{49}$
18. $\log_2(32) = x$
19. $\log_{11}(x) = 3$
20. $\log_5\left(\frac{1}{125}\right) = x$

Graph each of the following equations.

21. $y = \log(x + 2)$
22. $y = -5 + \log(x)$
23. $y = -\log(x - 4)$
24. $y = 5 + 3 \log(x - 7)$
Answers

1. \(4^y = x\)
2. \(2^3 = x\)
3. \(5^x = 30\)
4. \(\log_4(80) = x\)
5. \(\log_{1/2}(64) = x\)
6. \(\log_{343}(343) = 3\)
7. \(\log_5\left(\frac{1}{125}\right) = x\)
8. \(x^y = 32\)
9. \(\log_{11}(x) = 3\)
10. \(x^{-4} = \frac{1}{16}\)
11. \(x = 625\)
12. \(x = 81\)
13. \(x = 1,000,000,000\)
14. \(x = 2\)
15. \(x = -5\)
16. \(x = 5\)
17. \(x = -2\)
18. \(x = 5\)
19. \(x = 1331\)
20. \(x = -3\)
21. 
22. 
23. 
24. 

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