INTRODUCTION TO SEQUENCES

A.1.1 – A.1.3

In Appendix A, students investigate sequences by looking for patterns and rules. Initially in the appendix A, students concentrate on arithmetic sequences (sequences generated by adding a constant to the previous term), and then later in the Appendix A (and in Appendix B) consider geometric sequences (sequences generated by multiplying the previous term by a constant).

In Lessons A.1.1 through A.1.3 students are introduced to the two types of sequences, arithmetic and geometric, and their graphs, in everyday situations.

For additional examples and explanations, see the next section of this Parent Guide with Extra Practice, “Equations for Sequences.” For additional information, see the first half of the Math Notes box in Lesson A.3.2.

Example 1

Peachy Orchard Developers are preparing land to create a large subdivision of single-family homes. They have already built 15 houses on the site. Peachy Orchard plans to build six new homes every month. Create a table of values that will show the number of houses in the Peachy Orchard subdivision over time. Write an equation relating the number of houses over time. Graph the sequence.

Since the subdivision initially has 15 homes, 15 will be the amount at time \( t = 0 \). After one month, there will be six more, or 21, homes. After the second month, there will be 27 homes. After each month, we add six homes to the total number of homes in the subdivision. Because we are adding a constant amount after each time period, this is an arithmetic sequence.

We can find the equation for this situation by noticing that this is a linear function: the growth is constant. All arithmetic sequences are linear.

One way to write the equation that models this situation is to notice that the growth = slope = 6 homes/month, and the starting point = y-intercept = 15. Then in \( y = mx + b \) form, the equation is \( y = 6x + 15 \).

Another way to find the equation of a line, especially in situations more complex than this one, is to use two points on the line, calculate the slope \( (m) \) between the two points, and then solve for the y-intercept (as in Lesson 2.3.2). This method is shown in the following steps:

<table>
<thead>
<tr>
<th>( n ), the number of months</th>
<th>( t(n) ), the total number of homes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>1</td>
<td>21</td>
</tr>
<tr>
<td>2</td>
<td>27</td>
</tr>
<tr>
<td>3</td>
<td>33</td>
</tr>
<tr>
<td>4</td>
<td>39</td>
</tr>
</tbody>
</table>

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Choose \((1, 21)\) and \((4, 39)\)

\[
slope = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x} = \frac{39 - 21}{4 - 1} = \frac{18}{3} = 6
\]

\[
y = mx + b
\]

at \((x, y) = (1, 21)\) and \(m = 6,\)

\[
21 = 6(1) + b
\]

\[
b = 15
\]

\[
y = 6x + 15
\]

We write the equation as \(t(n) = 6n + 15\) to show that this is an arithmetic sequence (as opposed to the linear function \(y = mx + b\) or \(f(x) = mx + b\) that will find the term \(t\), for any number \(n\). Let \(t(n)\) represent the number of houses, and \(n\) the number of months.

The sequence would be written: 21, 27, 33, 39, …. Note that sequences usually begin with the first term (in this case, the term for the first month, \(n = 1\)).

The graph for the sequence is shown at right. Note that it is linear, and that it starts with the point \((1, 21)\).
Example 2

When Rosa tripped and fell into a muddy puddle at lunch (she was so embarrassed!), she knew exactly what would happen: within ten minutes, the two girls who saw her fall would each tell four people what they had seen. Within the next ten minutes, those eight students would each tell four more people. Rosa knew this would continue until everyone in the entire school was talking about her muddy experience. If there are 2016 students in the school, how many “generations” of gossiping would it take before everyone in the school was talking about Rosa? How many minutes would it take? Graph the situation.

At time $t = 0$, only two people see Rosa trip and fall. After ten minutes, each of those two people would tell four people; there are eight students gossiping about Rosa. After another ten minutes, each of those eight students will gossip with four more students; there will be $8 \times 4 = 32$ students gossiping. For the third increment of ten minutes, each of the 32 students will gossip with 4 students; $32 \times 4 = 128$ students gossiping.

Each time we multiply the previous number of students by four we get the next number of students. This is an example of a **geometric sequence**, and the multiplier is four. We record this in a table as shown at right, with $n$ being the number of ten minute increments since Rosa’s fall, and $t(n)$ is the number of students discussing the incident at that time. By continuing the table, we note that at time $t = 6$, there will be 2048 students discussing the mishap. Since there are only 2016 students in the school, everyone is gossiping by the sixth ten-minute increment of time. Therefore, just short of 60 minutes, or a little before one hour, everyone knows about Rosa’s fall in the muddy puddle.

<table>
<thead>
<tr>
<th>$n$, Number of Ten Minute Increments</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>128</td>
</tr>
<tr>
<td>4</td>
<td>512</td>
</tr>
<tr>
<td>5</td>
<td>1024</td>
</tr>
<tr>
<td>6</td>
<td>2048</td>
</tr>
</tbody>
</table>

The graph is shown at right. A geometric relationship is not linear; it is exponential. In future lessons, students would write the sequence as 8, 32, 128, … . Note that sequences usually begin with the first term (in this case, the term for the first month, $n = 1$).
Problems

1. Find the missing terms for this sequence and an equation for \( t(n) \).

\[ \_, 15, 11, \_, 3 \]

2. For this sequence each term is \( \frac{1}{5} \) of the previous one. Work forward and backward to find the missing terms.

\[ \_, \_, \frac{2}{3}, \_, \_ \]

3. The 30th term of a sequence is 42. If each term in the sequence is four greater than the previous number, what is the first term?

4. The microscopic length of a crystalline structure grows so that each day it is 1.005 times as long as the previous day. If on the third day the structure was 12.5 nm long, write a sequence for how long it was on the first five days. (nm stands for nanometer, or \( 1 \times 10^{-9} \) meters.)

5. Davis loves to ride the mini-cars at the amusement park but riders must be no more than 125 cm tall. If on his fourth birthday he is 94 cm tall and grows approximately 5.5 cm per year, at what age will he no longer be able to go on the mini-car ride?

Answers

1. 19 and 8; \( t(n) = 23 - 4n \)

2. \( \frac{50}{3}, \frac{10}{3}, \frac{2}{3}, \frac{2}{15}, \frac{2}{75} \)

3. \( 42 - 29(4) = -74 \)

4. \( \approx 12.28, \approx 12.44, 12.5, \approx 12.56, \approx 12.63 \)

5. \( t(n) = 5.5n + 94 \), so solve \( 5.5n + 94 \leq 125 \). \( n \approx 5.64 \). At \( \approx 4 + 5.64 = 9.64 \) he will be too tall. Davis can continue to go on the ride until he is about \( 9 \frac{1}{2} \) years old.
In these lessons, students learn multiple representations for sequences: as a string of numbers, as a table, as a graph, and as an equation. Read more about writing equations for sequences in the Math Notes box in Lesson B.2.3.

In addition to the ways to write explicit equations for sequences, as explained in the Math Notes box in Lesson B.2.3, equations for sequences less-commonly are written recursively. An explicit formula tells exactly how to find any specific term in the sequence. A recursive formula names the first term (or any other term) and how to get from one term to the next. For an explanation of recursive sequences, see the Math Notes box in Lesson A.3.2. For additional examples and more practice, see the Checkpoint 4A materials in the student textbook.

Example 1

This is the same scenario as in Example 1 of the previous section, Introduction to Sequences.

Peachy Orchard Developers are preparing land to create a large subdivision of single-family homes. They have already built 15 houses on the site. Peachy Orchard plans to build six new homes every month. Write a sequence for the number of houses built, then write an equation for the sequence. Fully describe a graph of this sequence.

The sequence is 21, 27, 33, 39, …. Note that sequences usually begin with the first term, where the number of months $n = 1$.

The common difference is $m = 6$, and the zeroth term is $b = 15$. The equation can be written $t(n) = mn + b = 6n + 15$. Note that for a sequence, $t(n) =$ is used instead of $y =$, $t(n) =$ indicates the equation is for a discrete sequence, as opposed to a continuous function. Students compared sequences to functions in Lesson 5.3.3.

The equation could also have been written as $a_n = 6n + 15$.

The graph of the sequence is shown at right. There are no $x$- or $y$-intercepts. There is no point at $(0, 15)$ because sequences are usually written starting with the first term where $n = 1$. The domain consists of integers (whole numbers) greater than or equal to one. The range consists of the $y$-values of the points that follow the rule $t(n) = 6n + 15$ when $n \geq 1$. There are no asymptotes. The graph is linear and is shown at right. This graph is discrete (separate points). Note: The related function, $y = 6x + 15$, would have the domain of all real numbers (including fractions and negatives) and the graph would be a continuous connected line.
Example 2

This is the same scenario as in Example 2 of the previous section, *Introduction to Sequences*.

When Rosa tripped and fell into a muddy puddle at lunch (she was so embarrassed!), she knew exactly what would happen: within ten minutes, the two girls who saw her fall would each tell four people what they had seen. Within the next ten minutes, those eight students would each tell four more people. Rosa knew this would continue until everyone in the entire school was talking about her muddy experience. Write a sequence for the number of people who knew about Rosa mishap in ten-minute intervals, then write an equation for the sequence. Fully describe a graph of this sequence.

The multiplier is \( b = 4 \), and the zeroth term is \( a = 2 \). The equation can be written \( t(n) = ab^n = 2 \cdot 4^n \). The equation could also have been written as \( a_n = 2 \cdot 4^n \). (Later, in Appendix B, students will also learn “first term” notation for sequences, \( a_n = 8 \cdot 4^{(n-1)} \).

The sequence is: 8, 32, 128, 512, … . Note that the sequence is written starting with \( n = 1 \).

The graph of the sequence is to the right. There are no \( x \)- or \( y \)-intercepts. There is no point at (0, 2) because sequences are usually written starting with the first term where \( n = 1 \). The domain consists of integers (whole numbers) greater than or equal to one. The range consists of the \( y \)-values of the points that follow the rule \( t(n) = 2(4)^n \) when \( n \geq 1 \). The graph is exponential and is shown at right. There is no symmetry. This graph is discrete (separate points). (Note: The related function, \( y = 2 \cdot 4^n \) would have the domain of all real numbers (including fractions and negatives) and the graph would be a solid curve.)
Example 3

Consider the two sequences:

A: $-8, -5, -2, 1, \ldots$
B: $256, 128, 64, \ldots$

da. For each sequence, is it arithmetic, geometric, or neither? How can you tell? Explain completely.

b. What are the zeroth term and the generator for each sequence?

c. For each sequence, write an equation representing the sequence.

d. Is $378$ a term of sequence A? Justify your answer.

e. Is $\frac{1}{4}$ a term of sequence B? Justify your answer.

To determine the type of sequence for A and B above, we have to look at the growth of each sequence.

A: $-8, -5, -2, 1, \ldots$

\[
\begin{array}{cccccc}
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\
+3 & +3 & +3 & \end{array}
\]

Sequence A is made (generated) by adding three to each term to get the next term. When each term has a common difference (in this case, “+3”) the sequence is arithmetic.

Sequence B, however, is different. The terms do not have a common difference.

B: $256, 128, 64, \ldots$

\[
\begin{array}{cccccc}
\downarrow & \downarrow & \downarrow & \downarrow & \\
-128 & -64 & \end{array}
\]

Instead, these terms have a common ratio (multiplier). A sequence with a common ratio is a geometric sequence.

B: $256, 128, 64, \ldots$

\[
\begin{array}{cccccc}
\downarrow & \downarrow & \downarrow & \downarrow & \\
\frac{1}{2} \cdot \frac{1}{2} & \end{array}
\]

The first term for sequence A is $-8$, and has a generator or common difference of $+3$. Therefore the zeroth term is $-11$ (because $-11 + 3 = -8$). An arithmetic sequence has an equation of the form $t(n) = mn + b$ (or $a_n = mn + a_0$) where $m$ is the common difference, and $b$ is the initial value. For sequence A, the equation is $t(n) = 3n - 11$, for $n = 1, 2, 3, \ldots$

Example continues on next page →
Example continued from previous page.

For sequence B, the first term is 256 with a generator or common ratio of \( \frac{1}{2} \). Therefore the zeroth term is 512, because \( 512 \cdot \frac{1}{2} = 256 \). The general equation for a geometric sequence is \( t(n) = ab^n \) where \( a \) is the zeroth term, and \( b \) is the common ratio (multiplier). For sequence B, the equation is \( t(n) = 512 \left( \frac{1}{2} \right)^n \) for \( n = 1, 2, 3, \ldots \)

To check if 378 is a term in sequence A, we could list the terms of the sequence out far enough to check, but that would be time consuming. Instead, we will check if there is an integer \( n \) that solves \( t(n) = 3n - 11 = 378 \).

\[
\begin{align*}
3n - 11 &= 378 \\
3n &= 389 \\
\frac{389}{3} &= 129 \frac{2}{3}
\end{align*}
\]

When we solve, \( n \) is not a whole number, therefore 378 cannot be a term in the sequence.

Similarly, to check if \( \frac{1}{4} \) is a term in sequence B, we need to solve \( t(n) = 512 \left( \frac{1}{2} \right)^n = \frac{1}{4} \), and look for a whole number solution.

\[
\begin{align*}
512 \left( \frac{1}{2} \right)^n &= \frac{1}{4} \\
\frac{1}{512} \cdot 512 \left( \frac{1}{2} \right)^n &= \frac{1}{512} \cdot \frac{1}{4} \\
\left( \frac{1}{2} \right)^n &= \frac{1}{2048} \\
\left( \frac{1}{2} \right)^n &= \frac{1}{211} \\
2^n &= \frac{211}{1} \\
n &= 11
\end{align*}
\]

Although solving an equation like this is probably new for most students, they can solve this problem by using guess-and-check. Also, by writing both sides as a power of 2, students can see the solution easily.

Since the equation has a whole number solution, \( \frac{1}{4} \) is a term of sequence B.

That is, when \( n = 11 \), \( t(n) = \frac{1}{4} \).

Examples For Arithmetic Sequences

List the first five terms of each arithmetic sequence.

**Example 4** (An explicit formula)

\[
t(n) = 5n + 2
\]

\[
\begin{align*}
t(1) &= 5(1) + 2 = 7 \\
t(2) &= 5(2) + 2 = 12 \\
t(3) &= 5(3) + 2 = 17 \\
t(4) &= 5(4) + 2 = 22 \\
t(5) &= 5(5) + 2 = 27
\end{align*}
\]

The sequence is: 7, 12, 17, 22, 27, …

**Example 5** (A recursive formula)

\[
t(1) = 3, \ t(n + 1) = t(n) - 5
\]

\[
\begin{align*}
t(1) &= 3 \\
t(2) &= t(1) - 5 = 3 - 5 = -2 \\
t(3) &= t(2) - 5 = -2 - 5 = -7 \\
t(4) &= t(3) - 5 = -7 - 5 = -12 \\
t(5) &= t(4) - 5 = -12 - 5 = -17
\end{align*}
\]

The sequence is: 3, –2, –7, –12, –17, …
Example 6

Find an explicit and a recursive formula for the sequence: −2, 1, 4, 7, …

Explicit: \( m = 3, b = −5 \) so the equation is: \( t(n) = mn + b = 3n − 5 \)
Recursive: \( t(1) = −2, t(n + 1) = t(n) + 3 \)

Examples For Geometric Sequences

List the first five terms of each geometric sequence.

Example 7  (An explicit formula)  
\[
t(n) = 3 \cdot 2^{n-1}
\]
\[
t(1) = 3 \cdot 2^{1-1} = 3 \\
t(2) = 3 \cdot 2^{2-1} = 3 \cdot 2^1 = 6 \\
t(3) = 3 \cdot 2^{3-1} = 3 \cdot 2^2 = 12 \\
t(4) = 3 \cdot 2^{4-1} = 3 \cdot 2^3 = 24 \\
t(5) = 3 \cdot 2^{5-1} = 3 \cdot 2^4 = 48
\]
The sequence is: 3, 6, 12, 24, 48, …

Example 8  (A recursive formula)  
\[
t(1) = 8, \ t(n + 1) = t(n) \cdot \frac{1}{2}
\]
\[
t(1) = 8 \\
t(2) = t(1) \cdot \frac{1}{2} = 8 \cdot \frac{1}{2} = 4 \\
t(3) = t(2) \cdot \frac{1}{2} = 4 \cdot \frac{1}{2} = 2 \\
t(4) = t(3) \cdot \frac{1}{2} = 2 \cdot \frac{1}{2} = 1 \\
t(5) = t(4) \cdot \frac{1}{2} = 1 \cdot \frac{1}{2} = \frac{1}{2}
\]
The sequence is: 8, 4, 2, 1, \( \frac{1}{2} \), …

Example 9

Find an explicit and a recursive formula for the sequence: 81, 27, 9, 3, …

Explicit: \( a_1 = 81, b = \frac{1}{3} \) so the \( a_0 \) (the zeroth term) is found by \( a_0 = 81 \div \frac{1}{3} = 243 \) and the answer is: \( a_n = a_0 \cdot b^n = 243 \left( \frac{1}{3} \right)^n \), or alternatively, \( t(n) = 243 \cdot \frac{1}{3}^n \)
Recursive: \( t(1) = 81, \ t(n + 1) = t(n) \cdot \frac{1}{3} \)
Problems

Each of the functions listed below defines a sequence. List the first five terms of the sequence, and state whether the sequence is arithmetic, geometric, both, or neither.

1. \( t(n) = 5n + 2 \)  
2. \( s_n = 3 - 8n \)  
3. \( u(n) = 9n - n^2 \)  
4. \( t(n) = (-4)^n \)
5. \( s(n) = \left(\frac{1}{4}\right)^n \)  
6. \( u(n) = n(n + 1) \)  
7. \( t(n) = 8 \)  
8. \( s_n = \frac{3}{4}n + 1 \)

Identify each of the following sequences as arithmetic or geometric. Then write the equation that gives the terms of the sequence.

9. \( 48, 24, 12, 6, 3, \ldots \)  
10. \( -4, 3, 10, 17, 24, \ldots \)  
11. \( 43, 39, 35, 31, 27, \ldots \)  
12. \( \frac{2}{3}, \frac{1}{2}, \frac{3}{8}, \frac{9}{32}, \frac{27}{128}, \ldots \)  
13. \( 5, -5, 5, -5, 5, \ldots \)  
14. \( 10, 1, 0.1, 0.01, 0.001, \ldots \)

Graph the following two sequences on the same set of axes.

15. \( t(n) = -6n + 20 \)  
16. \( 1, 4, 16, 64, \ldots \)

17. Do the two sequences of the last two problems have any terms in common? Explain how you know.

18. Every year since 1548, the average height of a human male has increased slightly. The new height is 100.05% of what it was the previous year. If the average male’s height was 54 inches in 1548, what is the average height of a male in 2008?

19. Davis has $5.40 in his bank account on his fourth birthday. If his parents add $0.40 to his bank account every week, when will he have enough to buy the new Smokin’ Derby race car set which retails for $24.99?

20. Fully describe the graph of the sequence \( t(n) = -4n + 18 \).

Arithmetic Sequences

List the first five terms of each arithmetic sequence.

21. \( t(n) = 5n - 2 \)  
22. \( t(n) = -3n + 5 \)
23. \( t(n) = -15 + \frac{1}{2}n \)  
24. \( t(n) = 5 + 3(n - 1) \)
25. \( t(1) = 5, t(n + 1) = t(n) + 3 \)  
26. \( t(1) = 5, t(n + 1) = t(n) - 3 \)
27. \( t(1) = -3, t(n + 1) = t(n) + 6 \)  
28. \( t(1) = \frac{1}{2}, t(a + 1) = t(n) + \frac{1}{2} \)
Appendix A

Find the 30th term of each arithmetic sequence.

29. \( t(n) = 5n - 2 \)  
30. \( t(n) = -15 + \frac{1}{2}n \)  
31. \( t(31) = 53, \quad d = 5 \)  
32. \( t(1) = 25, \quad t(n + 1) = t(n) - 3 \)

For each arithmetic sequence, find an explicit and a recursive formula.

33. \( 4, 8, 12, 16, 20, \ldots \)  
34. \( -2, 5, 12, 19, 26, \ldots \)  
35. \( 27, 15, 3, -9, -21, \ldots \)  
36. \( 3, \frac{1}{3}, 3 \frac{2}{3}, 4, 4 \frac{1}{3}, \ldots \)

Sequences are graphed using points of the form: (term number, term value).
For example, the sequence 4, 9, 16, 25, 36, … would be graphed by plotting the points (1, 4), (2, 9), (3, 16), (4, 25), (5, 36), …. Sequences are graphed as points and not connected.

37. Graph the sequences from problems 1 and 2 above. What are the slopes of the lines determined by the points?

38. How do the slopes of the lines found in the previous problem relate to the sequences?

**Geometric Sequences**

List the first five terms of each geometric sequence.

39. \( t(n) = 5 \cdot 2^n \)  
40. \( t(n) = -3 \cdot 3^n \)  
41. \( t(n) = 40 \left( \frac{1}{2} \right)^{n-1} \)  
42. \( t(n) = 6 (\frac{1}{2})^{n-1} \)  
43. \( t(1) = 5, \quad t(n + 1) = t(n) \cdot 3 \)  
44. \( t(1) = 100, \quad t(n + 1) = t(n) \cdot \frac{1}{2} \)  
45. \( t(1) = -3, \quad t(n + 1) = t(n) \cdot (-2) \)  
46. \( t(1) = \frac{1}{3}, \quad t(n + 1) = t(n) \cdot \frac{1}{2} \)

Find the 15th term of each geometric sequence.

47. \( t(14) = 232, \quad r = 2 \)  
48. \( t(16) = 32, \quad r = 2 \)  
49. \( t(14) = 9, \quad r = \frac{2}{3} \)  
50. \( t(16) = 9, \quad r = \frac{2}{3} \)

Find an explicit and a recursive formula for each geometric sequence.

51. \( 2, 10, 50, 250, 1250, \ldots \)  
52. \( 16, 4, 1, \frac{1}{4}, \frac{1}{16}, \ldots \)  
53. \( 5, 15, 45, 135, 405, \ldots \)  
54. \( 3, -6, 12, -24, 48, \ldots \)  
55. Graph the sequences from problems 39 and 52. Remember the note before problem 37 about graphing sequences.

56. How are the graphs of geometric sequences different from the graphs of arithmetic sequences?
Answers

1. 7, 12, 17, 22, 27, arithmetic, common difference is 5.
2. −5, −13, −21, −29, −37, arithmetic, common difference is −8.
3. 8, 14, 18, 20, 20, neither
4. −4, 16, −64, 256, −1024, geometric, common ratio is −4.
5. $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, $\frac{1}{256}$, $\frac{1}{1024}$, geometric, common ratio is $\frac{1}{4}$.
6. 2, 6, 12, 20, 30, neither
7. 8, 8, 8, 8, 8, both, common difference is 0, common ratio is 1.
8. $\frac{7}{4}$, $\frac{5}{2}$, $\frac{13}{4}$, 4, $\frac{19}{4}$, arithmetic, common difference is $\frac{3}{4}$.
9. geometric, $t(n) = 96\left(\frac{1}{2}\right)^n$
10. arithmetic, $t(n) = 7n - 11$
11. arithmetic, $t(n) = -4n + 47$
12. geometric, $t(n) = \left(\frac{8}{9}\right)\left(\frac{3}{4}\right)^n$
13. geometric, $t(n) = -5(-1)^n$
14. geometric, $t(n) = 100\left(\frac{1}{10}\right)^n$
15. See dots on graph.
16. See circles on graph.
17. No, they do not. The graph is discrete, or just points, which are the terms of each sequence. Since they do not share a common point, they do not have any terms in common.
18. In 2008, $54(1.0005)^{450} \approx 67.96$ inches.
19. $t(n) = 0.4n + 5.4$, so solve $0.4n + 5.4 \geq 24.99$. $n = 48.975$. In 49 weeks he will have $25. If he must also cover tax, he will need another three or four weeks.
20. This is a function, it represents an arithmetic sequence, the graph is discrete but the points are collinear. The y-intercept is (0, 18), there is no x-intercept. The domain is the numbers (1, 2, 3, ...), the range is the sequence itself: 14, 10, 6, 2, −2, ... There are no asymptotes.
21. \[3, 8, 13, 18, 23\]  
22. \[2, -1, -4, -7, -10\]  
23. \[-14\frac{1}{2}, -14, -13\frac{1}{2}, -13, -12\frac{1}{2}\]  
24. \[5, 8, 11, 14, 17\]  
25. \[5, 8, 11, 14, 17\]  
26. \[5, 2, -1, -4, -7\]  
27. \[-3, 3, 9, 15, 21\]  
28. \[\frac{1}{3}, \frac{5}{6}, 1\frac{1}{3}, 1\frac{5}{6}, 2\frac{1}{3}\]  
29. 148  
30. 0  
31. 48  
32. -62  
33. \(t(n) = 4n; \ t(1) = 4, \ t(n + 1) = t(n) + 4\)  
34. \(t(n) = 7n - 9; \ t(1) = -2, \ t(n + 1) = t(n) + 7\)  
35. \(t(n) = 39 - 12n; \ t(1) = 27, \ t(n + 1) = t(n) - 12\)  
36. \(a_n = \frac{1}{3}n + 2 \frac{2}{3}; \ a_1 = 3, \ a_{n+1} = a_n + \frac{1}{3}\)  
37. Graph (1): (1, 3), (2, 8), (3, 13), (4, 18), (5, 23) slope = 5  
Graph (2): (1, 2), (2, -1), (3, -4), (4, -7), (5, -10) slope = -3  
38. The slope of the line containing the points is the same as the common difference of the sequence.  
39. \[10, 20, 40, 80, 160\]  
40. \[-9, -27, -81, -243, -729\]  
41. \[40, 20, 10, 5, \frac{5}{2}\]  
42. \[6, -3, \frac{3}{2}, -\frac{3}{4}, \frac{3}{8}\]  
43. \[5, 15, 45, 135, 405\]  
44. \[100, 50, 25, \frac{25}{2}, \frac{25}{4}\]  
45. \[-3, 6, -12, 24, -48\]  
46. \[\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \frac{1}{48}\]  
47. 464  
48. 16  
49. 6  
50. \[\frac{27}{2}\]  
51. \(t(n) = \frac{2}{3} \cdot 5^n; \ t(1) = 2, \ t(n + 1) = t(n) \cdot 5\)  
52. \(t(n) = 64 \cdot \left(\frac{1}{4}\right)^n; \ t(1) = 16, \ t(n + 1) = t(n) \cdot \frac{1}{4}\)  
53. \(t(n) = \frac{5}{3} \cdot 3^n; \ t(1) = 5, \ t(n + 1) = t(n) \cdot 3\)  
54. \(t(n) = -\frac{3}{2} \cdot (-2)^n; \ t(1) = 3, \ t(n + 1) = t(n) \cdot (-2)\)  
55. Graph (39): (1, 10), (2, 20), (3, 40), (4, 80), (5, 160)  
Graph (52): (1, 16), (2, 4), (3, 1), (4, \(\frac{1}{4}\)), (5, \(\frac{1}{16}\))  
56. Arithmetic sequences are linear and geometric sequences are curved (exponential).
To recognize if a function is linear, exponential, or neither, look at the differences of the $y$-values between consecutive integer $x$-values. If the difference is constant, the graph is linear. If the difference is not constant, look at the pattern in the $y$-values. If a constant multiplier can be used to move from one $y$-value to the next, then the function is exponential. (Note that the same multiplier can be used to move from the difference to difference in an exponential function.)

Examples

Based on each table, identify the shape of the graph.

Example 1

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-3$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$-7$</td>
<td>$-5$</td>
<td>$-3$</td>
<td>$-1$</td>
<td>$1$</td>
<td>$3$</td>
<td>$5$</td>
</tr>
</tbody>
</table>

The difference in $y$-values is always two, a constant. The function is linear; the graph at right confirms this.

Example 2

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-3$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$9$</td>
<td>$4$</td>
<td>$1$</td>
<td>$0$</td>
<td>$1$</td>
<td>$4$</td>
<td>$9$</td>
</tr>
</tbody>
</table>

The first difference in $y$-values is not constant, and there is not a constant multiplier in moving from one $y$-value to the next. The function is neither linear nor exponential; the graph at right confirms this.

Example 3

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-3$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{2}$</td>
<td>$1$</td>
<td>$2$</td>
<td>$4$</td>
<td>$8$</td>
</tr>
</tbody>
</table>

The $y$-values have a constant multiplier of 2. (Also the differences in $y$-values have a constant multiplier of 2.) The function is exponential; the graph at right confirms this.
**Problems**

Based on the growth (the difference in y-values) shown in the tables, identify the corresponding graph as linear, exponential, or neither.

1. 
   \[
   \begin{array}{ccccccc}
   x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
   y & 14 & 10 & 6 & 2 & -2 & -6 & -10 \\
   \end{array}
   \]

2. 
   \[
   \begin{array}{ccccccc}
   x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
   y & \frac{1}{2} & 1 & 2 & 4 & 8 & 16 & 32 \\
   \end{array}
   \]

3. 
   \[
   \begin{array}{ccccccc}
   x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
   y & 21 & 12 & 5 & 0 & -3 & -4 & -3 \\
   \end{array}
   \]

4. 
   \[
   \begin{array}{ccccccc}
   x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
   y & -16 & -13 & -10 & -7 & -4 & -1 & 2 \\
   \end{array}
   \]

5. 
   \[
   \begin{array}{ccccccc}
   x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
   y & -14 & -9 & -4 & 1 & 6 & 11 & 16 \\
   \end{array}
   \]

6. 
   \[
   \begin{array}{ccccccc}
   x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
   y & -18 & -6 & -2 & 0 & 2 & 6 & 18 \\
   \end{array}
   \]

7. 
   \[
   \begin{array}{ccccccc}
   x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
   y & 4 & 8 & 16 & 32 & 64 & 128 & 256 \\
   \end{array}
   \]

8. 
   \[
   \begin{array}{ccccccc}
   x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
   y & \frac{1}{27} & \frac{1}{9} & \frac{1}{3} & 1 & 3 & 9 & 27 \\
   \end{array}
   \]

9. 
   \[
   \begin{array}{ccccccc}
   x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
   y & 30 & 20 & 12 & 6 & 2 & 0 & 0 \\
   \end{array}
   \]

10. 
    \[
    \begin{array}{ccccccc}
    x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
    y & 11 & 9 & 7 & 5 & 3 & 1 & -1 \\
    \end{array}
    \]

11. 
    \[
    \begin{array}{ccccccc}
    x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
    y & \frac{1}{9} & \frac{1}{3} & 1 & 3 & 9 & 27 & 81 \\
    \end{array}
    \]

12. 
    \[
    \begin{array}{ccccccc}
    x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
    y & \frac{1}{27} & \frac{1}{9} & \frac{1}{3} & 1 & 3 & 9 & 27 \\
    \end{array}
    \]

13. 
    \[
    \begin{array}{ccccccc}
    x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
    y & 0 & 5 & 8 & 9 & 8 & 5 & 0 \\
    \end{array}
    \]

14. 
    \[
    \begin{array}{ccccccc}
    x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
    y & 3 & 0 & -1 & 0 & 3 & 8 & 15 \\
    \end{array}
    \]

15. 
    \[
    \begin{array}{ccccccc}
    x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
    y & 1 & 0 & -1 & -2 & -1 & 0 & 1 \\
    \end{array}
    \]

16. 
    \[
    \begin{array}{ccccccc}
    x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
    y & \frac{9}{8} & \frac{9}{4} & \frac{9}{2} & 9 & 18 & 36 & 72 \\
    \end{array}
    \]
Answers

1. linear
2. exponential
3. neither
4. linear
5. linear
6. neither
7. exponential
8. exponential
9. neither
10. linear
11. exponential
12. neither
13. neither
14. neither
15. neither
16. exponential