Slope is a number that indicates the steepness (or flatness) of a line, as well as its direction (up or down) left to right. It is determined by the ratio between any two points on a line.

For lines that slope upward from left to right, the sign of the slope is positive. For lines that slope downward from left to right, the sign of the slope is negative.

Any linear equation written as $y = mx + b$, where $m$ and $b$ are any real numbers, is said to be in Slope-Intercept form. $m$ is the slope of the line. $b$ is the y-intercept, that is, the point $(0, b)$ is where the line intersects (crosses) the $y$-axis.

If two lines have the same slope, then they are parallel. Likewise, if two lines are parallel, then they have the same slope. That is, parallel lines have the same slope.

Two lines are perpendicular if the product of their slopes is $-1$. The slopes of perpendicular lines are the negative reciprocals of each other, that is, $m$ and $-\frac{1}{m}$.

Note that $m \cdot \left( \frac{-1}{m} \right) = -1$.

Examples: $3$ and $-\frac{1}{3}$, $-\frac{2}{3}$ and $\frac{3}{2}$, $\frac{5}{4}$ and $-\frac{4}{5}$

Two distinct lines on a flat surface that are not parallel intersect in a single point.

Example 1

Graph the linear equation $y = \frac{4}{7}x + 2$.

Using $y = mx + b$, the slope in $y = \frac{4}{7}x + 2$ is $\frac{4}{7}$ and the y-intercept is the point $(0, 2)$. To graph the line, begin at the y-intercept, $(0, 2)$. Remember that slope is so go up 4 units (since 4 is positive) from $(0, 2)$ and then move right 7 units. This gives a second point on the graph, $(7, 6)$. To create the graph, draw a straight line through the two points.
Example 2

A line has a slope of \( \frac{3}{4} \) and passes through the point (3, 2). What is the equation of the line?

Using \( y = mx + b \), write \( y = \frac{3}{4} x + b \). Since (3, 2) represents a point \((x, y)\) on the line, substitute 3 for \(x\) and 2 for \(y\), then solve for \(b\). This is shown in the work at right. Since \( b = -\frac{1}{4} \), the equation is \( y = \frac{3}{4} x - \frac{1}{4} \).

\[ 2 = \frac{3}{4} (3) + b \]
\[ 2 = \frac{9}{4} + b \]
\[ 2 - \frac{9}{4} = b \]
\[ b = -\frac{1}{4} \]

Example 3

Decide if the two lines at right are parallel, perpendicular, or neither (i.e., intersecting).

First find the slope of each equation. Then compare the slopes.

\[
\begin{align*}
5x - 4y &= -6 \\
-4y &= -5x - 6 \\
y &= \frac{5x + 6}{-4} \\
y &= \frac{5}{4}x + \frac{3}{2} \\

5y &= 4x + 3 \\
y &= \frac{4x + 3}{5} \\
y &= \frac{4}{5}x + \frac{3}{5}
\end{align*}
\]

The slope of this line is \( \frac{5}{4} \). The slope of this line is \( \frac{4}{5} \).

These two slopes are not equal, so they are not parallel. They intersect. The product of the two slopes is 1, not \(-1\), so they are not perpendicular.

Example 4

Write the equations of the lines through the given point, one parallel and one perpendicular to the given line \( y = -\frac{5}{2} x + 5 \) and point \((-4, 5)\).

For the parallel line, use \( y = mx + b \) with the same slope to write \( y = -\frac{5}{2} x + b \).

\[
\begin{align*}
5 &= -\frac{5}{2} (-4) + b \\
5 &= 10 + b \\
b &= -5
\end{align*}
\]

Therefore the equation of the parallel line through \((-4, 5)\) is \( y = -\frac{5}{2} x - 5 \).

For the perpendicular line, use \( y = mx + b \) where \( m \) is the negative reciprocal of the slope of the original equation to write \( y = \frac{2}{5} x + b \).

\[
\begin{align*}
5 &= \frac{2}{5} (-4) + b \\
5 &= -\frac{8}{5} + b \\
b &= \frac{33}{5}
\end{align*}
\]

Therefore the equation of the perpendicular line through \((-4, 5)\) is \( y = \frac{2}{5} x + \frac{33}{5} \).
Problems

Identify the \( y \)-intercept in each equation.

1. \( y = \frac{1}{2} x - 2 \)  
2. \( y = -\frac{3}{5} x - \frac{5}{3} \)  
3. \( 3x + 2y = 12 \)
4. \( x - y = -13 \)  
5. \( 2x - 4y = 12 \)  
6. \( 4y - 2x = 12 \)

Write the equation of the line with:

7. A slope = \( \frac{1}{2} \) and passing through the point \((4, 3)\).
8. A slope = \( \frac{2}{3} \) and passing through the point \((-3, -2)\).
9. A slope = \( -\frac{1}{3} \) and passing through the point \((4, -1)\).
10. A slope = \(-4\) and passing through the point \((-3, 5)\).

Determine the slope of each line using the highlighted points.

11.  
12.  
13.  

Using the slope and \( y \)-intercept, determine the equation of the line.

14.  
15.  
16.  
17.  

Graph the following linear equations on graph paper.

18. \( y = \frac{1}{2} x + 3 \)  
19. \( y = -\frac{3}{5} x - 1 \)  
20. \( y = 4x \)
21. \( y = -6x + \frac{1}{2} \)  
22. \( 3x + 2y = 12 \)
State whether each pair of lines is parallel, perpendicular, or neither.

23. \( y = 2x - 2 \) and \( y = 2x + 4 \)
24. \( y = \frac{1}{2}x + 3 \) and \( y = -2x - 4 \)
25. \( x - y = 2 \) and \( x + y = 3 \)
26. \( y - x = -1 \) and \( y + x = 3 \)
27. \( x + 3y = 6 \) and \( y = -\frac{1}{3}x - 3 \)
28. \( 3x + 2y = 6 \) and \( 2x + 3y = 6 \)
29. \( 4x = 5y - 3 \) and \( 4y = 5x + 3 \)
30. \( 3x - 4y = 12 \) and \( 4y = 3x + 7 \)

Write an equation for the line through the given point and parallel to the given line.

31. \( y = 2x - 2 \) and \((-3, 5)\)
32. \( y = \frac{1}{2}x + 3 \) and \((-4, 2)\)
33. \( x - y = 2 \) and \((-2, 3)\)
34. \( y - x = -1 \) and \((-2, 1)\)
35. \( x + 3y = 6 \) and \((-1, 1)\)
36. \( 3x + 2y = 6 \) and \((2, -1)\)
37. \( 4x = 5y - 3 \) and \((1, -1)\)
38. \( 3x - 4y = 12 \) and \((4, -2)\)

Write an equation for the line through the given point and perpendicular to the given line.

39. \( y = 2x - 2 \) and \((-3, 5)\)
40. \( y = \frac{1}{2}x + 3 \) and \((-4, 2)\)
41. \( x - y = 2 \) and \((-2, 3)\)
42. \( y - x = -1 \) and \((-2, 1)\)
43. \( x + 3y = 6 \) and \((-1, 1)\)
44. \( 3x + 2y = 6 \) and \((2, -1)\)
45. \( 4x = 5y - 3 \) and \((1, -1)\)
46. \( 3x - 4y = 12 \) and \((4, -2)\)

Write an equation for the line parallel to each line below through the given point.

47. Write the equation of the line passing through the point \((7, -8)\) which is parallel to the line through the points \((2, 5)\) and \((8, -3)\).
48. Write the equation of the line passing through the point \((1, -4)\) which is parallel to the line through the points \((-3, -7)\) and \((4, 3)\).
Answers

1. (0, –2)  
2. (0, –5/3)  
3. (0, 6)  
4. (0, 13)  
5. (0, –3)  
6. (0, 6)  
7. \( y = \frac{1}{2} x + 1 \)  
8. \( y = \frac{2}{3} x \)  
9. \( y = -\frac{1}{3} x + \frac{1}{3} \)  
10. \( y = -4x - 7 \)  
11. \(-\frac{1}{2}\)  
12. \( \frac{3}{4} \)  
13. –2  
14. \( y = 2x - 2 \)  
15. \( y = -x + 2 \)  
16. \( y = \frac{1}{3} x + 2 \)  
17. \( y = -2x + 4 \)  
18. slope = \( \frac{1}{2} \),  
   y-intercept (0, 3)  
19. slope = \( -\frac{3}{5} \),  
   y-intercept (0, –1)  
20. slope = 4,  
   y-intercept (0, 0)  
21. slope = –6,  
   y-intercept (0, \( \frac{1}{2} \))  
22. slope = \( -\frac{3}{2} \),  
   y-intercept (0, 6)  
23. parallel  
24. perpendicular  
25. perpendicular  
26. perpendicular  
27. parallel  
28. intersecting  
29. intersecting  
30. parallel  
31. \( y = 2x + 11 \)  
32. \( y = \frac{1}{2} x + 4 \)  
33. \( y = x + 5 \)  
34. \( y = x + 3 \)  
35. \( y = \frac{1}{3} x + \frac{2}{3} \)  
36. \( y = -\frac{3}{2} x + 2 \)  
37. \( y = \frac{3}{4} x - \frac{9}{5} \)  
38. \( y = \frac{3}{4} x - 5 \)  
39. \( y = -\frac{1}{2} x + \frac{7}{2} \)  
40. \( y = -2x - 6 \)  
41. \( y = -x + 1 \)  
42. \( y = -x - 1 \)  
43. \( y = 3x + 4 \)  
44. \( y = \frac{2}{3} x - \frac{7}{3} \)  
45. \( y = -\frac{5}{4} x + \frac{1}{4} \)  
46. \( y = -\frac{4}{3} x + \frac{10}{3} \)  
47. \( y = 3x + 11 \)  
48. \( y = -\frac{1}{2} x + \frac{15}{2} \)  
49. \( y = -\frac{4}{3} x + \frac{4}{3} \)  
50. \( y = \frac{10}{7} x - \frac{38}{7} \)
Use the Quadratic Formula to solve each of the following equations.

1. \( x^2 - x - 6 = 0 \) 
2. \( x^2 + 8x + 15 = 0 \) 
3. \( x^2 + 13x + 42 = 0 \) 
4. \( x^2 - 10x + 16 = 0 \) 
5. \( x^2 + 5x + 4 = 0 \) 
6. \( x^2 - 9x + 18 = 0 \) 
7. \( 5x^2 - x - 4 = 0 \) 
8. \( 4x^2 - 11x - 3 = 0 \) 
9. \( 6x^2 - x - 15 = 0 \) 
10. \( 6x^2 + 19x + 15 = 0 \) 
11. \( 3x^2 + 5x - 28 = 0 \) 
12. \( 2x^2 - x - 14 = 0 \) 
13. \( 4x^2 - 9x + 4 = 0 \) 
14. \( 2x^2 - 5x + 2 = 0 \) 
15. \( 20x^2 + 20x = 1 \) 
16. \( 13x^2 - 16x = 4 \) 
17. \( 7x^2 + 28x = 0 \) 
18. \( 5x^2 = -125x \) 
19. \( 8x^2 - 50 = 0 \) 
20. \( 15x^2 = 3 \)

**Answers**

1. \( x = -2, 3 \) 
2. \( x = -5, -3 \) 
3. \( x = -7, -6 \) 
4. \( x = 2, 8 \) 
5. \( x = -4, -1 \) 
6. \( x = 3, 6 \) 
7. \( x = -\frac{4}{5}, 1 \) 
8. \( x = -\frac{1}{4}, 3 \) 
9. \( x = -\frac{3}{2}, \frac{5}{3} \) 
10. \( x = -\frac{3}{2}, -\frac{5}{3} \) 
11. \( x = -4, \frac{2}{3} \) 
12. \( x = \frac{1 + \sqrt{113}}{4} \) 
13. \( x = \frac{9 + \sqrt{17}}{8} \) 
14. \( x = 2, \frac{1}{2} \) 
15. \( x = -\frac{20 + 4\sqrt{480}}{40}, -\frac{5 + \sqrt{30}}{10} \) 
16. \( x = \frac{16 + \sqrt{464}}{26}, \frac{8 + 2\sqrt{29}}{13} \) 
17. \( x = -4, 0 \) 
18. \( x = -25, 0 \) 
19. \( x = -\frac{5}{2}, \frac{5}{2} \) 
20. \( x = \pm\frac{\sqrt{5}}{5} \)