**Slope** is a number that indicates the steepness (or flatness) of a line, as well as its direction (up or down) left to right. It is determined by the ratio \( \frac{\text{vertical change}}{\text{horizontal change}} \) between any two points on a line.

For lines that slope upward from left to right, the sign of the slope is **positive**. For lines that slope downward from left to right, the sign of the slope is **negative**.

Any linear equation written as \( y = mx + b \), where \( m \) and \( b \) are any real numbers, is said to be in **Slope-Intercept form**. \( m \) is the slope of the line. \( b \) is the **y-intercept**, that is, the point \((0, b)\) is where the line intersects (crosses) the \( y \)-axis.

If two lines have the same slope, then they are **parallel**. Likewise, if two lines are parallel, then they have the same slope. That is, parallel lines have the same slope.

Two lines are **perpendicular** if the product of their slopes is \(-1\). The slopes of perpendicular lines are the negative reciprocals of each other, that is, \( m \) and \( -\frac{1}{m} \).

Note that \( m \cdot \left( -\frac{1}{m} \right) = -1 \).

Examples: \( 3 \) and \( -\frac{1}{3} \), \( -\frac{2}{3} \) and \( \frac{3}{2} \), \( \frac{5}{4} \) and \( -\frac{4}{5} \)

Two distinct lines on a flat surface that are not parallel intersect in a single point.

---

**Example 1**

Graph the linear equation \( y = \frac{4}{7}x + 2 \).

Using \( y = mx + b \), the slope in \( y = \frac{4}{7}x + 2 \) is \( \frac{4}{7} \) and the \( y \)-intercept is the point \((0, 2)\). To graph the line, begin at the \( y \)-intercept, \((0, 2)\). Remember that slope is \( \frac{\text{vertical change}}{\text{horizontal change}} \) so go up 4 units (since 4 is positive) from \((0, 2)\) and then move right 7 units. This gives a second point on the graph, \((7, 6)\). To create the graph, draw a straight line through the two points.
Example 2

A line has a slope of $\frac{3}{4}$ and passes through the point $(3, 2)$. What is the equation of the line?

Using $y = mx + b$, write $y = \frac{3}{4}x + b$. Since $(3, 2)$ represents a point $(x, y)$ on the line, substitute 3 for $x$ and 2 for $y$, then solve for $b$. This is shown in the work at right. Since $b = -\frac{1}{4}$, the equation is $y = \frac{3}{4}x - \frac{1}{4}$.

Example 3

Decide if the two lines at right are parallel, perpendicular, or neither (i.e., intersecting).

First find the slope of each equation. Then compare the slopes.

\[
\begin{align*}
5x - 4y &= -6 \\
-4y &= -5x - 6 \\
y &= \frac{5x - 6}{4} \\
y &= \frac{5}{4} x + \frac{3}{2} \\
\text{The slope of this line is } \frac{5}{4}. \\
\end{align*}
\]

\[
\begin{align*}
-4x + 5y &= 3 \\
5y &= 4x + 3 \\
y &= \frac{4x + 3}{5} \\
y &= \frac{4}{5} x + \frac{3}{5} \\
\text{The slope of this line is } \frac{4}{5}. \\
\end{align*}
\]

These two slopes are not equal, so they are not parallel. They intersect. The product of the two slopes is 1, not $-1$, so they are not perpendicular.

Example 4

Write the equations of the lines through the given point, one parallel and one perpendicular to the given line $y = -\frac{5}{2}x + 5$ and point $(-4, 5)$.

For the parallel line, use $y = mx + b$ with the same slope to write $y = -\frac{5}{2}x + b$.

\[
\begin{align*}
5 &= -\frac{5}{2}(-4) + b \\
5 &= \frac{20}{2} + b \\
5 &= 20 + b \\
b &= -5 \\
\end{align*}
\]

Therefore the equation of the parallel line through $(-4, 5)$ is $y = -\frac{5}{2}x - 5$.

For the perpendicular line, use $y = mx + b$ where $m$ is the negative reciprocal of the slope of the original equation to write $y = \frac{2}{5}x + b$.

\[
\begin{align*}
5 &= \frac{2}{5}(-4) + b \\
5 &= -\frac{8}{5} + b \\
5 &= \frac{-8}{5} + b \\
b &= \frac{33}{5} \\
\end{align*}
\]

Therefore the equation of the perpendicular line through $(-4, 5)$ is $y = \frac{2}{5}x + \frac{33}{5}$. $b = \frac{33}{5}$.
Problems

Identify the y-intercept in each equation.

1. \( y = \frac{1}{2} x - 2 \)
2. \( y = -\frac{3}{5} x - \frac{2}{3} \)
3. \( 3x + 2y = 12 \)
4. \( x - y = -13 \)
5. \( 2x - 4y = 12 \)
6. \( 4y - 2x = 12 \)

Write the equation of the line with:

7. A slope = \( \frac{1}{2} \) and passing through the point (4, 3).
8. A slope = \( \frac{2}{3} \) and passing through the point (–3, –2).
9. A slope = \( -\frac{1}{3} \) and passing through the point (4, –1).
10. A slope = –4 and passing through the point (–3, 5).

Determine the slope of each line using the highlighted points.

11. [Diagram]
12. [Diagram]
13. [Diagram]

Using the slope and y-intercept, determine the equation of the line.

14. [Diagram]
15. [Diagram]
16. [Diagram]
17. [Diagram]

Graph the following linear equations on graph paper.

18. \( y = \frac{1}{2} x + 3 \)
19. \( y = -\frac{3}{5} x - 1 \)
20. \( y = 4x \)
21. \( y = -6x + \frac{1}{2} \)
22. \( 3x + 2y = 12 \)
State whether each pair of lines is parallel, perpendicular, or neither.

23. \( y = 2x - 2 \) and \( y = 2x + 4 \)
24. \( y = \frac{1}{2}x + 3 \) and \( y = -2x - 4 \)
25. \( x - y = 2 \) and \( x + y = 3 \)
26. \( y - x = -1 \) and \( y + x = 3 \)
27. \( x + 3y = 6 \) and \( y = -\frac{1}{3}x - 3 \)
28. \( 3x + 2y = 6 \) and \( 2x + 3y = 6 \)
29. \( 4x = 5y - 3 \) and \( 4y = 5x + 3 \)
30. \( 3x - 4y = 12 \) and \( 4y = 3x + 7 \)

Write an equation for the line through the given point and parallel to the given line.

31. \( y = 2x - 2 \) and \((-3, 5)\)
32. \( y = \frac{1}{2}x + 3 \) and \((-4, 2)\)
33. \( x - y = 2 \) and \((-2, 3)\)
34. \( y - x = -1 \) and \((-2, 1)\)
35. \( x + 3y = 6 \) and \((-1, 1)\)
36. \( 3x + 2y = 6 \) and \((2, -1)\)
37. \( 4x = 5y - 3 \) and \((1, -1)\)
38. \( 3x - 4y = 12 \) and \((4, -2)\)

Write an equation for the line through the given point and perpendicular to the given line.

39. \( y = 2x - 2 \) and \((-3, 5)\)
40. \( y = \frac{1}{2}x + 3 \) and \((-4, 2)\)
41. \( x - y = 2 \) and \((-2, 3)\)
42. \( y - x = -1 \) and \((-2, 1)\)
43. \( x + 3y = 6 \) and \((-1, 1)\)
44. \( 3x + 2y = 6 \) and \((2, -1)\)
45. \( 4x = 5y - 3 \) and \((1, -1)\)
46. \( 3x - 4y = 12 \) and \((4, -2)\)

Write an equation for the line parallel to each line below through the given point.

47. \( y = 2x - 2 \) and \((-3, 5)\)
48. \( y = \frac{1}{2}x + 3 \) and \((-4, 2)\)

49. Write the equation of the line passing through the point \((7, -8)\) which is parallel to the line through the points \((2, 5)\) and \((8, -3)\).

50. Write the equation of the line passing through the point \((1, -4)\) which is parallel to the line through the points \((-3, -7)\) and \((4, 3)\).

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Answers

1. \((0, -2)\)  
2. \((0, -\frac{5}{3})\)  
3. \((0, 6)\)  
4. \((0, 13)\)  
5. \((0, -3)\)  
6. \((0, 6)\)  
7. \(y = \frac{1}{2}x + 1\)  
8. \(y = \frac{2}{3}x\)  
9. \(y = -\frac{1}{3}x + \frac{1}{3}\)  
10. \(y = -4x - 7\)  
11. \(-\frac{1}{2}\)  
12. \(\frac{3}{4}\)  
13. \(-2\)  
14. \(y = 2x - 2\)  
15. \(y = -x + 2\)  
16. \(y = \frac{1}{3}x + 2\)  
17. \(y = -2x + 4\)  
18. slope = \(\frac{1}{2}\), y-intercept \((0, 3)\)  
19. slope = \(-\frac{3}{5}\), y-intercept \((0, -1)\)  
20. slope = 4, y-intercept \((0, 0)\)  
21. slope = \(-6\), y-intercept \((0, \frac{1}{2})\)  
22. slope = \(-\frac{3}{2}\), y-intercept \((0, 6)\)  
23. parallel  
24. perpendicular  
25. perpendicular  
26. perpendicular  
27. parallel  
28. intersecting  
29. intersecting  
30. parallel  
31. \(y = 2x + 11\)  
32. \(y = \frac{1}{2}x + 4\)  
33. \(y = x + 5\)  
34. \(y = x + 3\)  
35. \(y = \frac{1}{3}x + \frac{2}{3}\)  
36. \(y = -\frac{3}{2}x + 2\)  
37. \(y = \frac{4}{2}x - \frac{9}{5}\)  
38. \(y = \frac{3}{4}x - 5\)  
39. \(y = -\frac{1}{2}x + \frac{7}{2}\)  
40. \(y = -2x - 6\)  
41. \(y = -x + 1\)  
42. \(y = -x - 1\)  
43. \(y = 3x + 4\)  
44. \(y = \frac{2}{3}x - \frac{7}{3}\)  
45. \(y = -\frac{3}{2}x + \frac{1}{4}\)  
46. \(y = -\frac{4}{3}x + \frac{10}{3}\)  
47. \(y = 3x + 11\)  
48. \(y = -\frac{1}{2}x + \frac{15}{2}\)  
49. \(y = -\frac{4}{3}x + \frac{4}{3}\)  
50. \(y = \frac{10}{7}x - \frac{38}{7}\)