SOLVING LINEAR SYSTEMS

You can find where two lines intersect (cross) by using algebraic methods. The two most common methods are the Substitution and Elimination (also known as the addition) Methods.

Example 1

Solve the following system of equations at right by using the Substitution Method. Check your solution.

When solving a system of equations, you are solving to find the x- and y-values that result in true statements when you substitute them into both equations. Since both equations are in y-form (that is, solved for y), and we know y = y, we can substitute the right side of each equation for y in the simple equation y = y and write 5x + 1 = −3x − 15. Then solve for x as shown at right.

Remember you must find x and y. To find y, use either of the two original equations. Substitute the value of x into the equation and find the value of y.

The solution appears to be (−2, −9). In order for this to be a solution, it must make both equations true when you replace x with −2 and y with −9. Substitute the values in both equations to check.

Therefore, (−2, −9) is the solution.

Example 2

Substitution can also be used when the equations are not in y-form.

Solve the system of equations at right by using the Substitution Method. Check your solution.

Rewrite the two equations as 4(−3y + 1) − 3y = −11 by replacing x with (−3y + 1), then solve for y as shown at right below.

Substitute y = 1 into x = −3y + 1. Solve for x, and write the answer for x and y as an ordered pair, (1, −2). Substitute y = 1 into 4x − 3y = −11 to verify that either original equation may be used to find the second coordinate. Check your answer as shown in Example 1.
Example 3

When you have a pair of two-variable equations, sometimes it is easier to **eliminate** one of the variables to obtain one single variable equation. You can do this by adding the two equations together as shown in the example below.

Solve the system at right. \[ \begin{align*}
2x + y &= 11 \\
x - y &= 4
\end{align*} \]

To eliminate the \( y \)-terms, add the two equations together then solve for \( x \).

\[
\begin{align*}
2x + y &= 11 \\
+ x - y &= 4 \\
3x &= 15 \\
x &= 5
\end{align*}
\]

Once we know the \( x \)-value we can substitute it into either of the original equations to find the corresponding value of \( y \). The first equation is shown at right.

Check the solution by substituting both the \( x \)-value and \( y \)-value into the other original equation.

\[
\begin{align*}
x - y &= 4 \\
5 - 1 &= 4 \\
4 &= 4 \quad \text{Check!}
\end{align*}
\]

Example 4

You can solve the system of equations at right by using the Elimination Method, but before you can eliminate one of the variables, you must adjust the coefficients of one of the variables so that they are additive opposites.

\[
\begin{align*}
3x + 2y &= 11 \\
4x + 3y &= 14
\end{align*}
\]

To eliminate \( y \), multiply the first equation by 3, then multiply the second equation by \(-2\) to get the equations at right.

\[
\begin{align*}
9x + 6y &= 33 \\
-8x - 6y &= -28
\end{align*}
\]

Next eliminate the \( y \)-terms by adding the two new equations.

\[
\begin{align*}
9x + 6y &= 33 \\
+ -8x - 6y &= -28 \\
x &= 5
\end{align*}
\]

Since \( x = 5 \), substitute 5 for \( x \) in either original equation to find that \( y = -2 \). The solution to the system of equations is \((5, -2)\).

You could also solve the system by multiplying the first equation by 4 and the second equation by \(-3\) to eliminate \( x \), then proceed as shown above to find \( y \).
Problems

Solve the following systems of equations to find the point of intersection \((x, y)\) for each pair of lines.

1. \(y = x - 6\)
   \(y = 12 - x\)
2. \(y = 3x - 5\)
   \(y = x + 3\)
3. \(x = 7 + 3y\)
   \(x = 4y + 5\)
4. \(x = -3y + 10\)
   \(x = -6y - 2\)
5. \(y = x + 7\)
   \(y = 4x - 5\)
6. \(y = 7 - 3x\)
   \(y = 2x - 8\)
7. \(y = 3x - 1\)
   \(2x - 3y = 10\)
8. \(x = -\frac{1}{2}y + 4\)
   \(8x + 3y = 31\)
9. \(2y = 4x + 10\)
   \(6x + 2y = 10\)
10. \(y = \frac{3}{2}x - 2\)
    \(y = \frac{1}{10}y + 1\)
11. \(y = 4x + 5\)
    \(y = x\)
12. \(4x - 3y = -10\)
    \(x = \frac{1}{4}y - 1\)
13. \(x + y = 12\)
    \(x - y = 4\)
14. \(2x - y = 6\)
    \(4x - y = 12\)
15. \(x + 2y = 7\)
    \(5x - 4y = 14\)
16. \(5x - 2y = 6\)
    \(4x + y = 10\)
17. \(x + y = 10\)
    \(x - 2y = 5\)
18. \(3y - 2x = 16\)
    \(y = 2x + 4\)
19. \(x + y = 11\)
    \(x = y - 3\)
20. \(x + 2y = 15\)
    \(y = x - 3\)
21. \(y + 5x = 10\)
    \(y - 3x = 14\)
22. \(y = 7x - 3\)
    \(4x + 2y = 8\)
23. \(y = 12 - x\)
    \(y = x - 4\)
24. \(y = 6 - 2x\)
    \(y = 4x - 12\)

Answers

1. \((9, 3)\)
2. \((4, 7)\)
3. \((13, 2)\)
4. \((22, -4)\)
5. \((4, 11)\)
6. \((3, -2)\)
7. \((-1, -4)\)
8. \((\frac{7}{2}, 1)\)
9. \((0, 5)\)
10. \((6, 1.6)\)
11. \((1, 1)\)
12. \((-0.25, 3)\)
13. \((8, 4)\)
14. \((3, 0)\)
15. \((4, 1.5)\)
16. \((2, 2)\)
17. \((\frac{25}{3}, \frac{5}{3})\)
18. \((1, 6)\)
19. \((4, 7)\)
20. \((7, 4)\)
21. \((-0.5, 12.5)\)
22. \((\frac{7}{9}, \frac{23}{9})\)
23. \((8, 4)\)
24. \((3, 0)\)