To graph a linear inequality, first graph the line for the corresponding equality. This line is known as the boundary line, since all the points that make the inequality true lie on one side or the other of the line. Before you draw the boundary line, decide whether the it is part of the solution or not, that is, whether the line is solid or dashed. If the inequality symbol is either ≤ or ≥, then the boundary line is part of the inequality and it must be solid. If the symbol is either < or >, then the boundary line is not part of the inequality and it must be dashed.

Next, decide which side of the boundary line must be shaded to show the coordinates (points) on the graph that make the inequality true. To do this, choose a point not on the line. Substitute this point into the \textit{original} inequality. If the inequality is true for this point, then shade the graph on this side of the line. If the inequality is false for the point, then shade the graph on the \textit{opposite} side of the line.

Note: If the inequality is not in Slope-Intercept form and you have to solve it for \( y \), \textit{always} use the \textit{original} inequality to test a point, \textit{not} the \( y \)-form form.

\textbf{Example 1}

Graph the inequality \( y > 3x - 2 \).

First, graph the line \( y = 3x - 2 \), but draw it dashed since \( > \) means the boundary line is not part of the solution.

Next, test the point \((-2, 4)\) to the left of the boundary line.

\[ 4 > 3(-2) - 2, \text{ so } 4 > -8 \]

Since the inequality is \textit{true} for this point, shade the graph on the left side of the boundary line.
Example 2

Graph the system of inequalities

\[ y \leq \frac{1}{2} x + 2 \quad \text{and} \quad y > -\frac{2}{3} x - 1 \]

Graph the lines \( y = -\frac{2}{3} x + 4 \)
and \( y = -\frac{2}{3} x - 1 \). The first line
is solid, the second is dashed.

Test the point \((-4, 5)\) in the first inequality.

This inequality is false, so shade on the opposite side of the
boundary line, from \((-4, 5)\).

\[ 5 \leq \frac{1}{2} (-4) + 2, \text{ so } 5 \leq 0 \]

Test the same point in the second inequality.

This inequality is true, so shade on the same side of the
boundary line as \((-4, 5)\).

\[ 5 > -\frac{2}{3} (-4) - 1, \text{ so } 5 > \frac{5}{3} \]

The solution is the overlap of the two shaded regions shown by the darkest shading in the second
graph above right.

Problems

Graph each of the following inequalities on separate sets of axes.

1. \( y \leq 3x + 1 \)
2. \( y \geq 2x - 1 \)
3. \( y \geq -2x - 3 \)
4. \( y \leq -3x + 4 \)
5. \( y > 4x + 2 \)
6. \( y < 2x + 1 \)
7. \( y < -3x - 5 \)
8. \( y > -5x - 4 \)
9. \( y \leq 3 \)
10. \( y \geq -2 \)
11. \( x > 1 \)
12. \( x \leq 8 \)
13. \( y > \frac{2}{3} x + 8 \)
14. \( y \leq -\frac{2}{3} x + 3 \)
15. \( y \leq -\frac{2}{7} x - 7 \)
16. \( y \geq \frac{1}{4} x - 2 \)
17. \( 3x + 2y \geq 7 \)
18. \( 2x - 3y \leq 5 \)
19. \( -4x + 2y < 3 \)
20. \( -3x - 4y > 4 \)

Graph each of the following pairs of inequalities on the same set of axes.

21. \( y > 3x - 4 \) and \( y \leq -2x + 5 \)
22. \( y \geq -3x - 6 \) and \( y > 4x - 4 \)
23. \( y \leq -\frac{3}{5} x + 4 \) and \( y \leq \frac{1}{3} x + 3 \)
24. \( y < -\frac{3}{7} x - 1 \) and \( y > \frac{4}{5} x + 1 \)
25. \( y < 3 \) and \( y \leq -\frac{1}{2} x + 2 \)
26. \( x \leq 3 \) and \( y < \frac{3}{4} x - 4 \)
Write an inequality for each of the following graphs.

27. [Graph]
28. [Graph]
29. [Graph]
30. [Graph]
31. [Graph]
32. [Graph]

Answers

1. [Graph]
2. [Graph]
3. [Graph]
4. [Graph]
5. [Graph]
6. [Graph]
7.  
8.  
9.  
10.  
11.  
12.  
13.  
14.  
15.  
16.  
17.  
18.
27. \( y \leq x + 5 \)  
28. \( y \geq -\frac{5}{2}x + 5 \)  
29. \( y \leq \frac{1}{2}x + 4 \)  
30. \( y > -4x + 1 \)  
31. \( x \geq -4 \)  
32. \( y \leq 3 \)