THE QUADRATIC FORMULA

You have used factoring and the Zero Product Property to solve quadratic equations. You can solve \textit{any} quadratic equation by using the \textbf{Quadratic Formula}.

If \(ax^2 + bx + c = 0\), then \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\).

For example, suppose \(3x^2 + 7x - 6 = 0\). Here \(a = 3\), \(b = 7\), and \(c = -6\).

Substituting these values into the formula results in:

\[
x = \frac{-7 \pm \sqrt{7^2 - 4(3)(-6)}}{2(3)} \Rightarrow x = \frac{-7 \pm \sqrt{121}}{6} \Rightarrow x = \frac{-7 \pm 11}{6}
\]

Remember that taking the square root of \(x^2\) yields both a positive and a negative square root. The sign \(\pm\) represents this fact for the square root in the formula and allows us to write the equation \textit{once} (representing two possible solutions) until later in the solution process.

Split the numerator into the two values: \(x = \frac{-7 + 11}{6}\) or \(x = \frac{-7 - 11}{6}\)

Thus the solution for the quadratic equation is: \(x = \frac{2}{3}\) or \(-3\)

\textbf{Example 1}

Solve \(x^2 + 3x - 2 = 0\) using the Quadratic Formula.

First, identify the values for \(a\), \(b\), and \(c\). In this case they are 1, 3, and \(-2\), respectively. Next, substitute these values into the Quadratic Formula.

\[
x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-2)}}{2(1)} \Rightarrow x = \frac{-3 \pm \sqrt{17}}{2}
\]

Then split the numerator into the two values: \(x = \frac{-3 + \sqrt{17}}{2}\) or \(x = \frac{-3 - \sqrt{17}}{2}\)

Using a calculator, the solution for the quadratic equation is: \(x \approx 0.56\) or \(-3.56\)
Example 2

Solve \(4x^2 + 4x = 3\) using the Quadratic Formula.

To solve any quadratic equation it must first be equal to zero. Rewrite the equation as \(4x^2 + 4x - 3 = 0\). Identify the values for \(a\), \(b\), and \(c\): 4, 4, and \(-3\), respectively.

Substitute these values into the Quadratic Formula:

\[
x = \frac{-4 \pm \sqrt{4^2 - 4(4)(-3)}}{2(4)} \Rightarrow x = \frac{-4 \pm \sqrt{64}}{8} \Rightarrow x = \frac{-4 \pm 8}{8}
\]

Split the numerator into the two values:

\[
x = \frac{-4 + 8}{8} \quad \text{or} \quad x = \frac{-4 - 8}{8}, \quad \text{so} \quad x = \frac{1}{2} \quad \text{or} \quad -\frac{3}{2}.
\]

Problems

Use the Quadratic Formula to solve each of the following equations.

1. \(x^2 - x - 6 = 0\)  
2. \(x^2 + 8x + 15 = 0\)  
3. \(x^2 + 13x + 42 = 0\)

4. \(x^2 - 10x + 16 = 0\)  
5. \(x^2 + 5x + 4 = 0\)  
6. \(x^2 - 9x + 18 = 0\)

7. \(5x^2 - x - 4 = 0\)  
8. \(4x^2 - 11x - 3 = 0\)  
9. \(6x^2 - x - 15 = 0\)

10. \(6x^2 + 19x + 15 = 0\)  
11. \(3x^2 + 5x - 28 = 0\)  
12. \(2x^2 - x - 14 = 0\)

13. \(4x^2 - 9x + 4 = 0\)  
14. \(2x^2 - 5x + 2 = 0\)  
15. \(20x^2 + 20x = 1\)

16. \(13x^2 - 16x = 4\)  
17. \(7x^2 + 28x = 0\)  
18. \(5x^2 = -125x\)

19. \(8x^2 - 50 = 0\)  
20. \(15x^2 = 3\)

Answers

1. \(x = -2, 3\)  
2. \(x = -5, -3\)  
3. \(x = -7, -6\)  
4. \(x = 2, 8\)

5. \(x = -4, -1\)  
6. \(x = 3, 6\)  
7. \(x = -\frac{4}{3}, 1\)  
8. \(x = -\frac{1}{4}, 3\)

9. \(x = \frac{-3 + \sqrt{5}}{2}, \frac{-3 + \sqrt{5}}{2}\)  
10. \(x = \frac{-3}{2}, \frac{-3}{2}\)  
11. \(x = -4, \frac{7}{3}\)  
12. \(x = \frac{1 + \sqrt{113}}{4}\)

13. \(x = \frac{9 + \sqrt{17}}{8}\)  
14. \(x = 2, \frac{1}{2}\)  
15. \(x = \frac{-20 + \sqrt{480}}{40}, \frac{-5 + \sqrt{30}}{10}\)

16. \(x = \frac{16 + \sqrt{464}}{26}, \frac{8 + 2\sqrt{79}}{13}\)  
17. \(x = -4, 0\)  
18. \(x = -25, 0\)

19. \(x = \frac{-5 + \sqrt{5}}{2}\)  
20. \(x = \frac{\pm \sqrt{5}}{2}\)