In the sequence 2, 6, 18, 54, …, there is a common ratio \( r = 3 \) between the successive terms and this is called an **geometric sequence**. There are two common methods to define a geometric sequence. The explicit formula tells you exactly how to find any specific term in the sequence. The recursive formula gives first term and how to get from one term to the next. Formally, for geometric sequences, this is written:

**Explicit:** \( a_n = a_1 \cdot r^{n-1} \) where \( n \) = term number and \( r \) = common ratio.

**Recursive:** \( a_1 = \) some specific value and \( a_{n+1} = a_n \cdot r \) where \( r \) = common ratio.

For the sequence 2, 6, 18, 54, …, the explicit formula is \( a_n = 2 \cdot 3^{n-1} \), and the recursive formula is \( a_1 = 2, \ a_{n+1} = a_n \cdot 3 \). In each case, successively replacing \( n \) by 1, 2, 3, … will yield the terms of the sequence. See the examples below.

**Example 1**

List the first five terms of the geometric sequence.

\[ a_n = 3 \cdot 2^{n-1} \] (an explicit formula)

\[ a_1 = 3 \cdot 2^{1-1} = 3 \cdot 2^0 = 3 \]
\[ a_2 = 3 \cdot 2^{2-1} = 3 \cdot 2^1 = 6 \]
\[ a_3 = 3 \cdot 2^{3-1} = 3 \cdot 2^2 = 12 \]
\[ a_4 = 3 \cdot 2^{4-1} = 3 \cdot 2^3 = 24 \]
\[ a_5 = 3 \cdot 2^{5-1} = 3 \cdot 2^4 = 48 \]

The sequence is: 3, 6, 12, 24, 48, …

**Example 2**

List the first five terms of the geometric sequence.

\[ b_1 = 8 \]
\[ b_{n+1} = b_n \cdot \frac{1}{2} \] (a recursive formula)

\[ b_1 = 8 \]
\[ b_2 = b_1 \cdot \frac{1}{2} = 8 \cdot \frac{1}{2} = 4 \]
\[ b_3 = b_2 \cdot \frac{1}{2} = 4 \cdot \frac{1}{2} = 2 \]
\[ b_4 = b_3 \cdot \frac{1}{2} = 2 \cdot \frac{1}{2} = 1 \]
\[ b_5 = b_4 \cdot \frac{1}{2} = 1 \cdot \frac{1}{2} = \frac{1}{2} \]

The sequence is: 8, 4, 2, 1, \( \frac{1}{2} \), …

**Example 3**

Write an explicit and a recursive equation for the sequence: 81, 27, 9, 3, …

Explicit: \( a_1 = 81, \ r = \frac{1}{3} \) so the equation is \( a_n = a_1 \cdot r^{n-1} = 81 \cdot \left( \frac{1}{3} \right)^{n-1} \).

Recursive: \( a_1 = 81, \ r = \frac{1}{3} \) so the equation is \( a_1 = 81, \ a_{n+1} = a_n \cdot \frac{1}{3} \).
Problems

List the first five terms of each geometric sequence.

1. \( a_n = 5 \cdot 2^{n-1} \)
2. \( b_n = -3 \cdot 3^{n-1} \)
3. \( a_n = 40 \left( \frac{1}{2} \right)^{n-1} \)
4. \( c_n = 6 \left( -\frac{1}{2} \right)^{n-1} \)
5. \( a_1 = 5, \ a_{n+1} = a_n \cdot 3 \)
6. \( a_1 = 100, \ a_{n+1} = a_n \cdot \frac{1}{2} \)
7. \( a_1 = -3, \ a_{n+1} = a_n \cdot (-2) \)
8. \( a_1 = \frac{1}{3}, \ a_{n+1} = a_n \cdot \frac{1}{2} \)

Find the 15th term of each geometric sequence.

9. \( b_{14} = 232, \ r = 2 \)
10. \( b_{16} = 32, \ r = 2 \)
11. \( a_{14} = 9, \ r = \frac{2}{3} \)
12. \( a_{16} = 9, \ r = \frac{2}{3} \)

Find an explicit and a recursive formula for each geometric sequence.

13. \( 2, 10, 50, 250, 1250, \ldots \)
14. \( 16, 4, 1, \frac{1}{4}, \frac{1}{16}, \ldots \)
15. \( 5, 15, 45, 135, 405, \ldots \)
16. \( 3, -6, 12, -24, 48, \ldots \)

17. Graph the sequences from problems 1 and 14.

18. How are the graphs of geometric sequences different from arithmetic sequences?

Answers

1. \( 5, 10, 20, 40, 80 \)
2. \( -3, -9, -27, -81, -243 \)
3. \( 40, 20, 10, 5, \frac{5}{2} \)
4. \( 6, -3, \frac{3}{2}, -\frac{3}{4}, \frac{3}{8} \)
5. \( 5, 15, 45, 135, 405 \)
6. \( 100, 50, 25, \frac{25}{2}, \frac{25}{4} \)
7. \( -3.6, -12, 24, -48 \)
8. \( \frac{1}{3} \cdot \frac{1}{6}, \frac{1}{12} \cdot \frac{1}{24}, \frac{1}{48} \)
9. \( 464 \)
10. \( 16 \)
11. \( 6 \)
12. \( \frac{27}{2} \)
13. \( a_n = 2 \cdot 5^{n-1}; \ a_1 = 2, \ a_{n+1} = a_n \cdot 5 \)
14. \( a_n = 16 \cdot \left( \frac{1}{4} \right)^{n-1}; \ a_1 = 16, \ a_{n+1} = a_n \cdot \frac{1}{4} \)
15. \( a_n = 5 \cdot 3^{n-1}; \ a_1 = 5, \ a_{n+1} = a_n \cdot 3 \)
16. \( a_n = 3 \cdot (-2)^{n-1}; \ a_1 = 3, \ a_{n+1} = a_n \cdot (-2) \)
17. Graph (1): (1, 5), (2, 10), (3, 20), (4, 40), (5, 80)
   Graph (14): (1, 16), (2, 4), (3, 1), (4, \frac{1}{4}), (5, \frac{1}{16})
18. Arithmetic sequences are linear and geometric sequences are curved (exponential).