There are two special right triangles that occur often in mathematics: the 30°-60°-90° triangle and the 45°-45°-90° triangle. By AA~, all 30°-60°-90° triangles are similar to each other, and all 45°-45°-90° triangles are similar to each other. Consequently, for each type of triangle, the sides are proportional. The sides of these triangles follow these patterns.

Another short cut in recognizing side lengths of right triangles are Pythagorean Triples. The lengths 3, 4, and 5 are sides of a right triangle (Note: You can verify this with the Pythagorean Theorem) and the sides of all triangles similar to the 3-4-5 triangle will have sides that form Pythagorean Triples (6-8-10, 9-12-15, etc). Another common Pythagorean Triple is 5-12-13.

See the Math Notes boxes in Lessons 5.2.1 and 5.3.1.

**Example 1**

The triangles below are either a 30°-60°-90° triangle or a 45°-45°-90° triangle. Decide which it is and find the lengths of the other two sides based on the pattern for that type of triangle.

a.  

```
\[ \begin{array}{c}
60° \\
60° \\
30° \\
30° \\
\end{array} \]
```

b.  

```
\[ \begin{array}{c}
60° \\
60° \\
14 \\
30° \\
\end{array} \]
```

c.  

```
\[ \begin{array}{c}
5 \\
5 \\
45° \\
45° \\
\end{array} \]
```

d.  

```
\[ \begin{array}{c}
8\sqrt{3} \\
8\sqrt{3} \\
30° \\
30° \\
\end{array} \]
```

e.  

```
\[ \begin{array}{c}
45° \\
45° \\
6\sqrt{2} \\
6\sqrt{2} \\
\end{array} \]
```

f.  

```
\[ \begin{array}{c}
45° \\
45° \\
10 \\
10 \\
\end{array} \]
```
In part (a), this is a 30°-60°-90° triangle, so its sides will fit the pattern for such a triangle. The pattern tells us that the hypotenuse is twice the length of the short leg. Since the short leg has a length of 6, the hypotenuse has a length of 12. The long leg is the length of the short leg times $\sqrt{3}$, so the long leg has a length of $6\sqrt{3}$.

In part (b), we have a 30°-60°-90° triangle again, but this time we know the length of the hypotenuse. Following the pattern, this means the length of the short leg is half the hypotenuse: 7. As before, we multiply the length of the short leg by $\sqrt{3}$ to get the length of the long leg: $7\sqrt{3}$.

The triangle in part (c) is a 45°-45°-90° triangle. The missing angle is also 45°; you can verify this by remembering the sum of the angles of a triangle is 180°. The legs of a 45°-45°-90° triangle are equal in length (it is isosceles) so the length of the missing leg is also 5. To find the length of the hypotenuse, we multiply the leg’s length by $\sqrt{2}$. Therefore the hypotenuse has length $5\sqrt{2}$.

We have another 30°-60°-90° triangle in part (d). This time we are given the length of the long leg. To find the short leg, we divide the length of the long leg by $\sqrt{3}$. Therefore, the length of the short leg is 8. To find the length of the hypotenuse, we double the length of the short leg, so the hypotenuse is 16.

The triangle in part (e) is a 45°-45°-90° triangle, and we are given the length of the hypotenuse. To find the length of the legs (which are equal in length), we will divide the length of the hypotenuse by $\sqrt{2}$. Therefore, each leg has length 6.

If you understand what was done in each of the previous parts, part (f) is no different from the rest. This is a 45°-45°-90° triangle, and we are given the length of the hypotenuse. However, we are used to seeing the hypotenuse of a 45°-45°-90° triangle with a $\sqrt{2}$ attached to it. In the last part when we were given the length of the hypotenuse, we divided by $\sqrt{2}$ to find the length of the legs, and this time we do the same thing.

$$\frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

Note: Multiplying by $\frac{\sqrt{2}}{\sqrt{2}}$ is called rationalizing the denominator. It is a technique to remove the radical from the denominator.
Example 2

Use what you know about Pythagorean Triples and similar triangles to fill in the missing lengths of sides below.

a. 

There are a few common Pythagorean Triples that students should recognize; 3–4–5, 5–12–13, 8–15–17, and 7–24–25 are the most common. If you forget about a particular triple or do not recognize one, you can always find the unknown side by using the Pythagorean Theorem if two of the sides are given. In part (a), this is a multiple of a 3–4–5 triangle. Therefore the length of the hypotenuse is 500. In part (b), we might notice that each leg has a length that is a multiply of four. Knowing this, we can rewrite them as $48 = 4(12)$, and $20 = 4(5)$. This is a multiple of a 5–12–13 triangle, the multiplier being 4. Therefore, the length of the hypotenuse is $4(13) = 52$. In part (c), do not let the decimal bother you. In fact, since we are working with Pythagorean Triples and their multiples, double both sides to create a similar triangle. This eliminates the decimal. That makes the leg 24 and the hypotenuse 25. Now we recognize the triple as 7–24-25. Since the multiple is 0.5, the length of the other leg is 3.5.
Problems

Identify the special triangle relationships. Then solve for \(x\), \(y\), or both.

1. \(x = 8\sqrt{3}, y = 8\)
2. \(x = y = 8\)
3. \(x = 13\)
4. \(y = 800\)
5. \(x = 6, y = 6\sqrt{2}\)
6. \(x = y = \frac{12}{\sqrt{2}} = 6\sqrt{2}\)
7. \(x = 11\sqrt{3}, y = 22\)
8. \(x = 45, y = 22.5\)
9. \(x = 34\)
10. \(x = 48\)