Two triangles are congruent if there is a sequence of rigid transformations that carry one onto the other. Two triangles are also congruent if they are similar figures with a ratio of similarity of 1, that is $\frac{1}{1}$. One way to prove triangles congruent is to prove they are similar first, and then prove that the ratio of similarity is 1. In these lessons, students find short cuts that enable them to prove triangles congruent in fewer steps, by developing five triangle congruence conjectures. They are SSS $\cong$, ASA $\cong$, AAS $\cong$, SAS $\cong$, and HL $\cong$, illustrated below.

Note: “S” stands for “side” and “A” stands for “angle.” HL $\cong$ is only used with right triangles. The “H” stands for “hypotenuse” and the “L” stands for leg. The pattern appears to be “SSA” but this arrangement is NOT one of our conjectures, since it is only true for right triangles.

See the Math Notes boxes in Lessons 6.1.1 and 6.1.4.
Example 1

Use your triangle congruence conjectures to decide whether or not each pair of triangles must be congruent. Base each decision on the markings, not on appearances. Justify each answer.

a. 

b. 

c. 

d. 

e. 

f. 

In part (a), the triangles are congruent by the SAS ≅ conjecture. The triangles are also congruent in part (b), this time by the SSS ≅ conjecture. In part (c), the triangles are congruent by the AAS ≅ conjecture. Part (d) shows a pair of triangles that are not necessarily congruent. The first triangle displays an ASA arrangement, while the second triangle displays an AAS arrangement. The triangles could still be congruent, but based on the markings, we cannot conclude that they definitely are congruent. The triangles in part (e) are right triangles and the markings fit the HL ≅ conjecture. Lastly, in part (f), the triangles are congruent by the ASA ≅ conjecture.
Example 2

Using the information given in the diagrams below, decide if any triangles are congruent, similar but not congruent, or not similar. If you claim the triangles are congruent or similar, create a flow chart justifying your answer.

In part (a), \( \triangle ABD \cong \triangle CBD \) by the SAS \( \cong \) conjecture. Note: If you only see “SA,” observe that \( BD \) is congruent to itself. The Reflexive Property justifies stating that something is equal or congruent to itself.

\[
\begin{align*}
\triangle ABD & \cong \triangle CBD \\
\text{Given} & \\
\text{SAS} & \\
\end{align*}
\]

In part (b), \( \triangle WXV \sim \triangle ZYV \) by the AA \( \sim \) conjecture. The triangles are not necessarily congruent; they could be congruent, but since we only have information about angles, we cannot conclude anything else.

\[
\begin{align*}
\angle WXV & \cong \angle YZV \\
\text{Vertical angles} & \\
\triangle WXV & \sim \triangle ZYV \\
\text{AA} & \\
\end{align*}
\]

There is more than one way to justify the answer to part (b). There is another pair of alternate interior angles (\( \angle WXV \) and \( \angle YZV \)) that are equal that we could have used rather than the vertical angles, or we could have used them along with the vertical angles.
Problems

Briefly explain if each of the following pairs of triangles are congruent or not. If so, state the triangle congruence conjecture that supports your conclusion.

1. 

2. 

3. 

4. 

5. 

6. 

7. 

8. 

9. 

10. 

11. 

12. 

13. 

14. 

15. 

16. 

17. 

Use your triangle congruence conjectures to decide whether or not each pair of triangles must be congruent. Base your decision on the markings, not on appearances. Justify your answer.
Using the information given in each diagram below, decide if any triangles are congruent, similar but not congruent, or not similar. If you claim the triangles are congruent or similar, create a flowchart justifying your answer.
In each diagram below, are any triangles congruent? If so, prove it. (Note: Justify some using flowcharts and some by writing two-column proofs.)

26.

27.

28.

29.

30.

31.

Complete a proof for each problem below in the style of your choice.

32. Given: $\overline{TR}$ and $\overline{MN}$ bisect each other.  
Prove: $\triangle NTP \cong \triangle MRP$

33. Given: $\overline{CD}$ bisects $\angle ACB$; $\angle 1 \equiv \angle 2$.  
Prove: $\triangle CDA \cong \triangle CDB$

34. Given: $\overline{AB} \parallel \overline{CD}$, $\angle B \equiv \angle D$, $\overline{AB} \equiv \overline{CD}$  
Prove: $\triangle ABF \cong \triangle CDE$

35. Given: $\overline{PG} \equiv \overline{SG}$, $\overline{TP} \equiv \overline{TS}$  
Prove: $\triangle TPG \equiv \triangle TSG$

36. Given: $\overline{OE} \perp \overline{MP}$, $\overline{OE}$ bisects $\angle MOP$  
Prove: $\triangle MOE \cong \triangle POE$

37. Given: $\overline{AD} \parallel \overline{BC}$, $\overline{DC} \parallel \overline{BA}$  
Prove: $\triangle ADB \cong \triangle CBD$

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38. Given: \( \overline{AC} \) bisects \( \overline{DE} \), \( \angle A \equiv \angle C \)
Prove: \( \triangle ADB \cong \triangle CEB \)

![Diagram of \( \triangle ADB \) and \( \triangle CEB \)]

39. Given: \( \overline{PQ} \perp \overline{RS} \), \( \angle R \equiv \angle S \)
Prove: \( \triangle PQR \cong \triangle PQS \)

![Diagram of \( \triangle PQR \) and \( \triangle PQS \)]

40. Given: \( \angle S \equiv \angle R \), \( \overline{PQ} \) bisects \( \angle SQR \)
Prove: \( \triangle SPQ \cong \triangle RPQ \)

![Diagram of \( \triangle SPQ \) and \( \triangle RPQ \)]

41. Given: \( \overline{TU} \equiv \overline{GY} \), \( \overline{KY} \parallel \overline{HU} \), \( \overline{KT} \perp \overline{TG} \), \( \overline{HG} \perp \overline{TG} \)
Prove: \( \angle K \equiv \angle H \)

![Diagram of \( \triangle TUH \) and \( \overline{KT} \) and \( \overline{HG} \)]

42. Given: \( \overline{MQ} \parallel \overline{WL} \), \( \overline{MQ} \equiv \overline{WL} \)
Prove: \( \overline{ML} \parallel \overline{WQ} \)

![Diagram of \( \overline{MQ} \parallel \overline{WL} \) and \( \overline{ML} \parallel \overline{WQ} \)]

Consider the diagram at right.

43. Is \( \triangle ABCD \equiv \triangle EDC \)? Prove it!

44. Is \( \overline{AB} \equiv \overline{DC} \)? Prove it!

45. Is \( \overline{AB} \equiv \overline{ED} \)? Prove it!
Answers

1. \( \triangle ABC \cong \triangle DEF \) by ASA.
2. \( \triangle GIH \cong \triangle LJK \) by SAS.
3. \( \triangle PNM \cong \triangle PNO \) by SSS.
4. \( \overline{QS} \cong \overline{QS} \), so \( \triangle QRS \cong \triangle QTS \) by HL.
5. The triangles are not necessarily congruent.
6. \( \triangle ABC \cong \triangle DFE \) by ASA or AAS.
7. \( \overline{GI} \cong \overline{GI} \), so \( \triangle GHI \cong \triangle IJG \) by SSS.

8. Alternate interior angles = used twice, so \( \triangle KLN \cong \triangle NMK \) by ASA.
9. Vertical angles \( \equiv \) at 0, so \( \triangle POQ \cong \triangle ROS \) by SAS.
10. Vertical angles and/or alternate interior angles =, so \( \triangle TUX \cong \triangle VWX \) by ASA.
11. No, the length of each hypotenuse is different.
12. Pythagorean Theorem, so \( \triangle EGH \cong \triangle IHG \) by SSS.
13. Sum of angles of triangle = 180\(^\circ\), but since the equal angles do not correspond, the triangles are not congruent.
14. \( AF + FC = FC + CD \), so \( \triangle ABC \cong \triangle DEF \) by SSS.
15. \( \overline{XZ} \cong \overline{XZ} \), so \( \triangle WXZ \cong \triangle YXZ \) by AAS.
16. \( \triangle ABC \cong \triangle EDC \) by AAS \( \equiv \)
17. \( \triangle PQS \cong \triangle PRS \) by AAS \( \equiv \), with \( \overline{PS} \cong \overline{PS} \) by the Reflexive Property.
18. \( \triangle VXW \cong \triangle ZXY \) by ASA \( \equiv \), with \( \angle VXW \cong \angle ZXY \) because vertical angles are \( \equiv \).
19. \( \triangle TEA \cong \triangle SAE \) by SSS \( \equiv \), with \( \overline{EA} \cong \overline{EA} \) by the Reflexive Property.
20. \( \triangle KLB \cong \triangle EBL \) by HL \( \equiv \), with \( \overline{BL} \cong \overline{BL} \) by the Reflexive Property.


22. \( \triangle DAV \sim \triangle ISV \) by SAS ~

23. \( \triangle LUN \) and \( \triangle HTC \) are not necessarily similar based on the markings.
24. \( \triangle SAP \sim \triangle SJE \) by AA

\[ \begin{array}{l}
\text{Alt. int. angles} = m\angle SAP = m\angle SJE \\
\text{Alt. int. angles} = m\angle SPA = m\angle SEJ \\
\triangle SAP \sim \triangle SJE \\
\end{array} \]

25. \( \triangle KRS \cong \triangle ISR \) by HL

\[ \begin{array}{l}
\triangle KRS & \cong \triangle ISR \\
KR = IS & \text{Given} \\
RS = RS & \text{Ref. Prop.} \\
\end{array} \]

26. Yes

\[ \begin{array}{l}
\angle BAD \cong \angle BCD & \text{Given} \\
\angle BDC \cong \angle BDA & \text{Right \( \angle s \) are \( \cong \)} \\
BD \cong BD & \text{Reflexive Prop.} \\
\triangle ABD \cong \triangle CBD & \text{AAS} \end{array} \]

27. Yes

\[ \begin{array}{l}
\angle B \cong \angle E & \text{Given} \\
\angle BCA \cong \angle ECD & \text{Vertical \( \angle s \) are \( \cong \)} \\
\angle BDE \cong \angle EDC & \text{ASA} \end{array} \]

28. Yes

\[ \begin{array}{l}
AC \cong CD & \text{Given} \\
\angle BDC \cong \angle BDA & \text{Right \( \angle s \) are \( \cong \)} \\
BC \cong BC & \text{Reflexive Prop.} \\
\triangle ABC \cong \triangle BDC & \text{SAS} \end{array} \]

29. Yes

\[ \begin{array}{l}
AD \cong CB & \text{Given} \\
CA \cong AC & \text{Reflexive Prop.} \\
\triangle ABC \cong \triangle ACD & \text{SSS} \end{array} \]

30. Not necessarily.

Counterexample:

31. Yes
32. \( \overline{NP} \equiv \overline{MP} \) and \( \overline{TP} \equiv \overline{RP} \) by definition of bisector. \( \angle NPT \equiv \angle MPR \) because vertical angles are equal. So, \( \Delta NTP \equiv \Delta MRP \) by SAS \( \cong \).

33. \( \angle ACD \equiv \angle BCD \) by definition of angle bisector. \( \overline{CD} \equiv \overline{CD} \) by reflexive so \( \Delta CDA \equiv \Delta CDB \) by ASA \( \cong \).

34. \( \angle A \equiv \angle C \) since alternate interior angles of parallel lines congruent so \( \Delta AFB \equiv \Delta CDE \) by ASA \( \cong \).

35. \( \overline{TG} \equiv \overline{TG} \) by reflexive so \( \Delta TPG \equiv \Delta TSG \) by SSS \( \cong \).

36. \( \angle MEO \equiv \angle PEO \) because perpendicular lines form \( \cong \) right angles \( \angle MOE \equiv \angle POE \) by angle bisector and \( \overline{OE} \equiv \overline{OE} \) by reflexive. So, \( \Delta MOE \equiv \Delta POE \) by ASA \( \cong \).

37. \( \angle CDB \equiv \angle ABD \) and \( \angle ADB \equiv \angle CBD \) since parallel lines give congruent alternate interior angles. \( \overline{DB} \equiv \overline{DB} \) by reflexive so \( \Delta ADB \equiv \Delta CBD \) by ASA \( \cong \).

38. \( \overline{DB} \equiv \overline{EB} \) by definition of bisector. \( \angle DBA \equiv \angle EBC \) since vertical angles are congruent. So \( \Delta ADB \equiv \Delta CEB \) by AAS \( \cong \).

39. \( \angle SQP \equiv \angle RQP \) since perpendicular lines form congruent right angles. \( \overline{PQ} \equiv \overline{PQ} \) (Reflexive Prop.) so \( \Delta PQR \equiv \Delta PQS \) by AAS \( \cong \).

40. \( \angle SQP \equiv \angle RQP \) by angle bisector and \( \overline{PQ} \equiv \overline{PQ} \) by reflexive, so \( \Delta SPQ \equiv \Delta RPQ \) by AAS \( \cong \).

41. \( \angle KYT \equiv \angle HUG \) because parallel lines form congruent alternate exterior angles. \( TY + YU = YU + GU \) so \( TY \equiv GU \) by subtraction. \( \angle T \equiv \angle G \) since perpendicular lines form congruent right angles. So \( \Delta KTY \equiv \Delta HGU \) by ASA \( \cong \). Therefore, \( \angle K \equiv \angle H \) since \( \equiv \) triangles have congruent parts.

42. \( \angle MQL \equiv \angle WLQ \) since parallel lines form congruent alternate interior angles. \( \overline{QL} \equiv \overline{QL} \) by the Reflexive Property so \( \Delta MQL \equiv \Delta WLQ \) by SAS \( \cong \), then \( \angle WQL \equiv \angle MLQ \) since congruent triangles have congruent parts. So \( \overline{ML} \parallel \overline{WQ} \) since congruent alternate interior angles are formed by parallel lines.

43. Yes

44. Not necessarily.

45. Not necessarily.