Two triangles are congruent if there is a sequence of rigid transformations that carry one onto the other. Two triangles are also congruent if they are similar figures with a ratio of similarity of 1, that is $\frac{1}{1}$. One way to prove triangles congruent is to prove they are similar first, and then prove that the ratio of similarity is 1. In these lessons, students find short cuts that enable them to prove triangles congruent in fewer steps, by developing five triangle congruence conjectures. They are SSS $\equiv$, ASA $\equiv$, AAS $\equiv$, SAS $\equiv$, and HL $\equiv$, illustrated below.

Note: “S” stands for “side” and “A” stands for “angle.” HL $\equiv$ is only used with right triangles. The “H” stands for “hypotenuse” and the “L” stands for leg. The pattern appears to be “SSA” but this arrangement is NOT one of our conjectures, since it is only true for right triangles.

See the Math Notes boxes in Lessons 6.1.1 and 6.1.4.
Example 1

Use your triangle congruence conjectures to decide whether or not each pair of triangles must be congruent. Base each decision on the markings, not on appearances. Justify each answer.

a. 

b. 

c. 

d. 

e. 

f. 

In part (a), the triangles are congruent by the SAS $\cong$ conjecture. The triangles are also congruent in part (b), this time by the SSS $\cong$ conjecture. In part (c), the triangles are congruent by the AAS $\cong$ conjecture. Part (d) shows a pair of triangles that are not necessarily congruent. The first triangle displays an ASA arrangement, while the second triangle displays an AAS arrangement. The triangles could still be congruent, but based on the markings, we cannot conclude that they definitely are congruent. The triangles in part (e) are right triangles and the markings fit the HL $\cong$ conjecture. Lastly, in part (f), the triangles are congruent by the ASA $\cong$ conjecture.
Example 2

Using the information given in the diagrams below, decide if any triangles are congruent, similar but not congruent, or not similar. If you claim the triangles are congruent or similar, create a flow chart justifying your answer.

a. ![Diagram](image1)

In part (a), \( \triangle ABD \cong \triangle CBD \) by the SAS \( \cong \) conjecture. Note: If you only see “SA,” observe that \( BD \) is congruent to itself. The **Reflexive Property** justifies stating that something is equal or congruent to itself.

![Flow Chart](image2)

In part (b), \( \triangle WXV \sim \triangle ZYV \) by the AA \( \sim \) conjecture. The triangles are not necessarily congruent; they could be congruent, but since we only have information about angles, we cannot conclude anything else.

![Flow Chart](image3)

There is more than one way to justify the answer to part (b). There is another pair of alternate interior angles (\( \angle WXV \) and \( \angle ZYV \)) that are equal that we could have used rather than the vertical angles, or we could have used them along with the vertical angles.
Problems

Briefly explain if each of the following pairs of triangles are congruent or not. If so, state the triangle congruence conjecture that supports your conclusion.

1. 

2. 

3. 

4. 

5. 

6. 

7. 

8. 

9. 

10. 

11. 

12. 

13. 

14. 

15. 

16. 

17. 

Use your triangle congruence conjectures to decide whether or not each pair of triangles must be congruent. Base your decision on the markings, not on appearances. Justify your answer.
18. Using the information given in each diagram below, decide if any triangles are congruent, similar but not congruent, or not similar. If you claim the triangles are congruent or similar, create a flowchart justifying your answer.

19.

20.

21.

22.

23.

24.

25.
In each diagram below, are any triangles congruent? If so, prove it. (Note: Justify some using flowcharts and some by writing two-column proofs.)

26. 

27. 

28. 

29. 

30. 

31. 

Complete a proof for each problem below in the style of your choice.

32. Given: $\overline{TR}$ and $\overline{MN}$ bisect each other.
Prove: $\triangle NTP \cong \triangle MRP$

33. Given: $\overline{CD}$ bisects $\angle ACB$; $\angle 1 \equiv \angle 2$.
Prove: $\triangle CDA \cong \triangle CDB$

34. Given: $\overline{AB} \parallel \overline{CD}$, $\angle B \equiv \angle D$, $\overline{AB} \equiv \overline{CD}$
Prove: $\triangle ABF \cong \triangle CDE$

35. Given: $\overline{PG} \equiv \overline{SG}$, $\overline{TP} \equiv \overline{TS}$
Prove: $\triangle TPG \cong \triangle TSG$

36. Given: $\overline{OE} \perp \overline{MP}$, $\overline{OE}$ bisects $\angle MOP$
Prove: $\triangle MOE \cong \triangle POE$

37. Given: $\overline{AD} \parallel \overline{BC}$, $\overline{DC} \parallel \overline{BA}$
Prove: $\triangle ADB \cong \triangle CBD$
38. Given: $AC$ bisects $DE$, $\angle A \equiv \angle C$  
Prove: $\triangle ADB \cong \triangle CEB$

![Diagram 38](image)

39. Given: $PQ \perp RS$, $\angle R \equiv \angle S$  
Prove: $\triangle PQR \cong \triangle PQS$

![Diagram 39](image)

40. Given: $\angle S \equiv \angle R$, $PQ$ bisects $\angle SQR$  
Prove: $\triangle SPQ \cong \triangle RPQ$

![Diagram 40](image)

41. Given: $TU \equiv GY$, $KY \parallel HU$, $KT \perp TG$, $HG \perp TG$. Prove: $\angle K \equiv \angle H$

![Diagram 41](image)

42. Given: $MQ \parallel WL$, $MQ \equiv WL$  
Prove: $ML \parallel WQ$

![Diagram 42](image)

Consider the diagram at right.

43. Is $\triangle BCD \equiv \triangle EDC$? Prove it!

44. Is $AB \equiv DC$? Prove it!

45. Is $AB \equiv ED$? Prove it!
Answers

1. \( \triangle ABC \cong \triangle DEF \) by ASA \( \cong \).
2. \( \triangle GIH \cong \triangle JK \) by SAS \( \cong \).
3. \( \triangle PNM \cong \triangle PNO \) by SSS \( \cong \).
4. \( \overline{QS} \cong \overline{QS} \), so \( \triangle QRS \cong \triangle QTS \) by HL \( \cong \).
5. The triangles are not necessarily congruent.
6. \( \triangle ABC \cong \triangle DFE \) by ASA \( \cong \) or AAS \( \cong \).
7. \( \overline{GI} \cong \overline{GI} \), so \( \triangle GHI \cong \triangle JI \) by SSS \( \cong \).
8. Alternate interior angles = used twice, so \( \triangle KLN \cong \triangle NMK \) by ASA \( \cong \).
9. Vertical angles \( \cong \) at 0, so \( \triangle POQ \cong \triangle ROS \) by SAS \( \cong \).
10. Vertical angles and/or alternate interior angles =, so \( \triangle TUX \cong \triangle VWX \) by ASA \( \cong \).
11. No, the length of each hypotenuse is different.
12. Pythagorean Theorem, so \( \triangle EGH \cong \triangle HIG \) by SSS \( \cong \).
13. Sum of angles of triangle = 180º, but since the equal angles do not correspond, the triangles are not congruent.
14. \( AF + FC = FC + CD \), so \( \triangle ABC \cong \triangle DEF \) by SSS \( \cong \).
15. \( \overline{XZ} \cong \overline{XZ} \), so \( \triangle WXZ \cong \triangle YXZ \) by AAS \( \cong \).
16. \( \triangle ABC \cong \triangle EDC \) by AAS \( \cong \).
17. \( \triangle PQS \cong \triangle PRS \) by AAS \( \cong \), with \( \overline{PS} \cong \overline{PS} \) by the Reflexive Property.
18. \( \triangle VXW \cong \triangle ZXY \) by ASA \( \cong \), with \( \angle VXW \cong \angle ZXY \) because vertical angles are \( \cong \).
19. \( \triangle TEA \cong \triangle SAE \) by SSS \( \cong \), with \( \overline{EA} \cong \overline{EA} \) by the Reflexive Property.
20. \( \triangle KLB \cong \triangle EBL \) by HL \( \cong \), with \( \overline{BL} \cong \overline{BL} \) by the Reflexive Property.
22. \( \triangle DAV \sim \triangle ISV \) by SAS \( \sim \)
23. \( \triangle LUN \) and \( \triangle HTC \) are not necessarily similar based on the markings.
24. \( \triangle SAP \sim \triangle SJE \) by AA ~

\[ AP \parallel JE \]

\[ \text{given} \]

Alt. int. angles =

\[ m\angle SAP = m\angle SJE \]

\[ m\angle SPA = m\angle SEJ \]

\[ \triangle SAP \sim \triangle SJE \]

25. \( \triangle KRS \cong \triangle ISR \) by HL \( \cong \)

\[ KR = IS \]

Given

\[ RS = RS \]

Refl. Prop.

\[ \triangle KRS \cong \triangle ISR \]

26. Yes

\[ \angle BAD \cong \angle BCD \]

\[ BD \cong BD \]

\[ BDC \cong BDA \]

\[ \triangle ABD \cong \triangle CBD \]

AAS \( \cong \)

27. Yes

\[ \angle B \cong \angle E \]

\[ BC \cong CE \]

\[ \angle BCA \cong \angle ECD \]

\[ \triangle ABD \cong \triangle DEC \]

ASA \( \cong \)

28. Yes

\[ AC \cong CD \]

\[ BC \cong BC \]

\[ \angle BDC \cong \angle BDA \]

\[ \triangle ABC \cong \triangle DBC \]

SAS \( \cong \)

29. Yes

\[ AD \cong CB \]

\[ CA \cong AC \]

\[ \triangle ABC \cong \triangle DCA \]

SSS \( \cong \)

30. Not necessarily. Counterexample:

31. Yes

\[ BC \cong EF \]

\[ AC \cong DF \]

\[ \triangle ABC \cong \triangle DEF \]

HL \( \cong \)

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32. \( \overline{NP} \cong \overline{MP} \) and \( \overline{TP} \cong \overline{RP} \) by definition of bisector. \( \angle NPT \cong \angle MPR \) because vertical angles are equal. So, \( \triangle NTP \cong \triangle MRP \) by SAS \( \cong \).

33. \( \angle ACD \cong \angle BCD \) by definition of angle bisector. \( \overline{CD} \cong \overline{CD} \) by reflexive so \( \triangle CDA \cong \triangle CDB \) by ASA \( \cong \).

34. \( \angle A \cong \angle C \) since alternate interior angles of parallel lines congruent so \( \triangle AFB \cong \triangle CDE \) by ASA \( \cong \).

35. \( \overline{TG} \cong \overline{TG} \) by reflexive so \( \triangle TPG \cong \triangle TSG \) by SSS \( \cong \).

36. \( \angle CDB \cong \angle ABD \) and \( \angle ADB \cong \angle CBD \) since parallel lines give congruent alternate interior angles. \( \overline{DB} \cong \overline{DB} \) by reflexive so \( \triangle ADB \cong \triangle CBD \) by ASA \( \cong \).

37. \( \angle MEO \cong \angle PEO \) because perpendicular lines form right angles \( \angle MOE \cong \angle POE \) by angle bisector and \( \overline{OE} \cong \overline{OE} \) by reflexive. So, \( \triangle MOE \cong \triangle POE \) by ASA \( \cong \).

38. \( \angle SQP \cong \angle RQP \) since perpendicular lines form congruent right angles. \( \overline{PQ} \cong \overline{PQ} \) (Reflexive Prop.) so \( \triangle PQR \cong \triangle PQS \) by AAS \( \cong \).

39. \( \angle SQP \cong \angle RQP \) by angle bisector and \( \overline{PQ} \cong \overline{PQ} \) by reflexive, so \( \triangle SPQ \cong \triangle RPQ \) by AAS \( \cong \).

40. \( \angle SQP \cong \angle RQP \) by angle bisector and \( \overline{PQ} \cong \overline{PQ} \) by reflexive, so \( \triangle SPQ \cong \triangle RPQ \) by AAS \( \cong \).

41. \( \angle KYT \cong \angle HUG \) because parallel lines form congruent alternate exterior angles. \( TY + YU = YU + GU \) so \( TY \cong GU \) by subtraction. \( \angle T \cong \angle G \) since perpendicular lines form congruent right angles. So \( \angle KTY \cong \angle HGU \) by ASA \( \cong \). Therefore, \( \angle K \cong \angle H \) since \( \cong \) triangles have congruent parts.

42. \( \angle MQL \cong \angle WLQ \) since parallel lines form congruent alternate interior angles. \( \overline{QL} \cong \overline{QL} \) by the Reflexive Property so \( \triangle MQL \cong \triangle WLQ \) by SAS \( \cong \), then \( \angle WQL \cong \angle MLQ \) since congruent triangles have congruent parts. So \( ML \parallel WQ \) since congruent alternate interior angles are formed by parallel lines.

43. Yes

44. Not necessarily.

45. Not necessarily.