CIRCLES

Circles have special properties. The fact that they can roll smoothly is because the circle has a constant **diameter** (the distance across the circle that passes through the center). A vehicle with square wheels would cause it to bump up and down because, since the diagonals of a square are longer then its width, it does not have a constant diameter. But a circle is not the only shape with a constant diameter. Reuleaux curves, which resemble rounded polygons, also have a constant diameter. It may not appear to be the case, but Reuleaux curves roll smoothly without bumping up and down. See problem 7-3 in the textbook for a picture.

A circle does not include its interior. It is the set of points on a flat surface at a fixed distance (the radius) from a fixed point (the center). This also means that the center, diameters, and radii (plural of radius) are not part of the circle. Remember, a radius is half of a diameter, and connects the center of the circle to a point on the circle. A circle has infinitely many diameters and infinitely many radii.

See the Math Notes box in Lesson 7.1.2.

**Example 1**

Using the circle at right, write an equation and solve for \( x \).

Note: Each part is a different problem.

**a.** \( AO = 3x - 4, \ OB = 4x - 12 \).

**b.** \( OB = 2x - 5, \ AC = x - 7 \)

Using the information we have about circles, diameters, and radii, we can write an equation using the expressions in part (a), then solve for \( x \). \( AO \) and \( OB \) are both radii of circle \( O \), which means that they are equal in length.

In part (b), \( OB \) is a radius, but \( AC \) is a diameter, so \( AC \) is twice as long as \( OB \).

\[
AO = OB \\
3x - 4 = 4x - 12 \\
8 = x
\]

Subtract 3x and add 12 on both sides.

\[
2(\text{OB}) = \text{AC} \\
2(2x - 5) = x - 7 \\
4x - 10 = x - 7 \\
3x = 3 \\
x = 1
\]
Problems

Using the circle below, write an equation and solve for $x$. Note: Each part is a different problem.

1. $OP = 5x - 3, \ OR = 3x + 9$
2. $OQ = 2x + 12, \ OP = 3x - 1$
3. $OR = 12x - 8, \ OQ = 8x - 4$
4. $OP = 5x + 3, \ PR = 3x + 13$
5. $OQ = x - 6, \ PR = x + 7$

Answers

1. $x = 6$
2. $x = 13$
3. $x = 1$
4. $x = 1$
5. $x = 19$