Through the use of rigid transformations (rotations, reflections, and translations), students created new figures and discovered properties of these figures. Students also used transformations to discover the shortest path from one object to another (an optimization problem), and to define and study regular polygons. A regular polygon is a polygon in which all sides have equal length and all angles have equal measure.

See the Math Notes boxes in Lesson 7.1.4.

Example 1

On a recent scout camping trip, Zevel was walking back to camp when he noticed that the campfire had grown too large. He wants to fill his bucket at the river, then walk to the fire to douse the flames. To ensure the fire does not get out of control, Zevel wants to take the shortest path. What is the shortest path from where Zevel is standing to go to the river and then to the fire?

The shortest distance between two points on a flat surface is a straight line. How can we find the shortest distance if a third point is involved, such as Zevel’s trip to the river? We use reflections to find the point at the river to which Zevel should walk. Reflecting across a line gives a new figure that is the same distance from the line as the original. Reflecting point $F$ across the river line gives $F'$ with $FX = F'X$. To find the shortest distance from $Z$ to $F'$ we connect them. Since $\angle FXR$ is a right angle, $\triangle FXR \cong \triangle F'XR$ by the SAS $\cong$ conjecture. This means that $FR = F'R$. Since the shortest path from $Z$ to $F'$ is the straight line drawn, then the shortest path from Zevel to the river and then to the fire is to walk from $Z$ to $R$ and then to $F$.

Example 2

If a hinged mirror is set at $10^\circ$ and the core region (the region the hinged mirror encloses on the paper) is isosceles, how many sides would there be on the polygon that would be reflected in the mirror?

If the core region is isosceles, the reflected image will be a regular polygon. In a regular polygon, all the interior angles are equal in measure and, if the center is connected to each vertex of the polygon, these central angles are equal. If one central angle measures $10^\circ$ and there are $360^\circ$ around the center, there are $360^\circ \div 10^\circ = 36$ sides on the polygon.
Problems

1. Venetia wants to install two lights in her garden. Each one will be connected to a control timer that will turn the lights on and off automatically. She can mount the timer anywhere on her house, but she wants to minimize the amount of wire she will use. If the wire must run from the light at point $P$ to the timer, and then back out to the light at point $Q$, where should Venetia place the timer?

2. While playing miniature golf last weekend, Myrtle came to the fifth hole and saw that it was a par 1 hole. This meant that she should be able to putt the ball into the hole with one stroke. Explain to Myrtle how knowledge of “shortest distance” problems can help her make the putt.

3. If the center of a regular dodecagon (12 sides) is connected to each vertex of the figure, what is the measure of each angle at the center?

4. If a central angle of a regular polygon measures $18^\circ$, how many sides does the polygon have?

5. The center point of a regular pentagon is connected to each vertex forming five congruent isosceles triangles. Find the measure of each base angle in the isosceles triangles and use that result to find the measure of one interior angle of the pentagon.

Answers

1. Venetia should place the timer about 11.54 feet from the point $X$ or 18.45 feet from the point $Y$ on the diagram above.

2. If Myrtle can aim correctly and hit a straight shot, she can make a hole in one. She can imagine the hole reflected across the top boundary to find the direction to aim. If she hits the wall at the point $X$, the ball will travel to the hole.

3. $30^\circ$

4. 20 sides

5. Each base angle measures $54^\circ$. Two together makes one interior angle of the pentagon, so an interior angle measures $108^\circ$. 