AREA RATIOS OF SIMILAR FIGURES 8.2.1 – 8.2.2

Students return to similarity once again to explore what happens to the area of a figure if it is reduced or enlarged. In Chapter 3, students learned about the ratio of similarity, also called the “zoom factor.” If two similar figures have a ratio of similarity of \( \frac{a}{b} \), then the ratio of their perimeters is also \( \frac{a}{b} \), while the ratio of their areas is \( \frac{a^2}{b^2} \).

See the Math Notes boxes in Lessons 8.2.1 and 9.1.5.

Example 1

The figures \( P \) and \( Q \) at right are similar.

a. What is the ratio of similarity?

b. What is the perimeter of figure \( P \)?

c. Use your previous two answers to find the perimeter of figure \( Q \).

d. If the area of figure \( P \) is 34 square units, what is the area of figure \( Q \)?

The ratio of similarity is the ratio of the lengths of two corresponding sides. In this case, since we only have the length of one side of figure \( Q \), we will use the side of \( P \) that corresponds to that side. Therefore, the ratio of similarity is \( \frac{3}{7} \).

To find the perimeter of figure \( P \), add up all the side lengths: \( 3 + 6 + 4 + 5 + 3 = 21 \). If the ratio of similarity of the two figures is \( \frac{3}{7} \) then ratio of their perimeters is \( \frac{3}{7} \) as well.

\[
\begin{align*}
\text{perimeter } P &= 3 + 6 + 4 + 5 + 3 = 21 \\
\text{perimeter } Q &= \frac{21}{3} = \frac{7}{3} \\
3Q &= 147 \\
\text{perimeter } Q &= 49
\end{align*}
\]

If the ratio of similarity is \( \frac{3}{7} \) then the ratio of the areas is \( \left( \frac{3}{7} \right)^2 = \frac{9}{49} \).

\[
\begin{align*}
\text{area } P &= \left( \frac{3}{7} \right)^2 \\
\text{area } Q &= \left( \frac{3}{7} \right)^2 \times \frac{34}{9} = \frac{9}{49} \\
9Q &= 1666 \\
\text{area } Q &= 185.11 \text{ square units}
\end{align*}
\]
Example 2

Two rectangles are similar. If the area of the first rectangle is 49 square units, and the area of the second rectangle is 256 square units, what is the ratio of similarity between these two rectangles?

Since the rectangles are similar, if the ratio of similarity is \( \frac{a}{b} \), then the ratio of their areas is \( \frac{a^2}{b^2} \). We are given the areas so we know the ratio of their areas is \( \frac{49}{256} \). Therefore we can write:

\[
\frac{a}{b} = \sqrt{\frac{49}{256}} = \frac{7}{16}.
\]

The ratio of similarity between the two rectangles is \( \frac{a}{b} = \frac{7}{16} \). This can be written as a decimal or as a fraction.

Problems

1. If figure A and figure B are similar with a ratio of similarity of \( \frac{5}{4} \), and the perimeter of figure A is 18 units, what is the perimeter of figure B?

2. If figure A and figure B are similar with a ratio of similarity of \( \frac{1}{8} \), and the area of figure A is 13 square units, what is the area of figure B?

3. If figure A and figure B are similar with a ratio of similarity of 6, that is, 6 to 1, and the perimeter of figure A is 54 units, what is the perimeter of figure B?

4. If figure A and figure B are similar and the ratio of their perimeters is \( \frac{17}{6} \), what is their ratio of similarity?

5. If figure A and figure B are similar and the ratio of their areas is \( \frac{32}{9} \), what is their ratio of similarity?

6. If figure A and figure B are similar and the ratio of their perimeters is \( \frac{23}{11} \), does that mean the perimeter of figure A is 23 units and the perimeter of figure B is 11 units? Explain.

Answers

1. 14.4 units  
2. 832 sq. units  
3. 9 units  
4. \( \frac{17}{6} \)  
5. \( \frac{\sqrt{32}}{\sqrt{9}} = \frac{5.66}{3} \approx 1.89 \)

6. No, it just tells us the ratio. Figure A could have a perimeter of 46 units while figure B has a perimeter of 22 units.