TRANSFORMING FUNCTIONS

In Lesson 10.2.1, students transform functions by adding “k” to a given function \( f(x) \). If the original function is \( f(x) \), then the transformed function \( f(x) + k \) is a vertical shift of the original function by \( k \) units. If \( k \) is positive, then the function is shifted upward. If \( k \) is negative, then the function is shifted downward.

Example 1

\( f(x) = 2x \)  
Graph \( f(x) + 3 \).

Start by making a table for \( f(x) \).

Add a column to the table for \( f(x) + 3 \).

Note that \( f(x) \) can be thought of as \( y \), so when completing the table for \( f(x) + 3 \), think of it as \( y + 3 \).

Graph \( f(x) \) and \( f(x) + 3 \) on the same set of axes.

In the graph at right, \( f(x) \) is graphed with a dashed line and \( f(x) + 3 \) is graphed with a solid line.

Example 2

\( f(x) = \left(\frac{1}{2}\right)^x \)  
Graph \( f(x) - 4 \).

Make a table similar to the one made in the previous example.

Graph \( f(x) \) and \( f(x) - 4 \) on the same set of axes.

In the graph at right, \( f(x) \) is graphed with a dashed curve and \( f(x) - 4 \) is graphed with a solid curve.
Problems

Graph the original function and the transformed function on the same set of axes.

1. \( f(x) = -2x \)  
2. \( f(x) = \frac{3}{4}x \)  
3. \( f(x) = 2^x \)  
4. \( f(x) = \left(\frac{2}{3}\right)^x \)  

\( f(x) + 3 \)  
\( f(x) - 1 \)  
\( f(x) + 1 \)  
\( f(x) - 5 \)

Explain how each transformation is different from \( f(x) \).

5. \( f(x) - 8 \)  
6. \( f(x) - 50 \)  
7. \( f(x) + 31 \)  
8. \( f(x) + 36 \)

Given the graph of \( f(x) \) at right, graph each transformation.

9. \( f(x) - 4 \)  
10. \( f(x) + 2 \)

Answers

In answer graphs for problems 1 through 4, the original function is graphed using a dotted line or curve and the transformed function is graphed using a solid line or curve.

5. Shifted down 8 units.  
6. Shifted down 50 units.  
7. Shifted up 31 units.  
8. Shifted up 36 units.  
9 and 10.