ARITHMETIC OPERATIONS WITH FUNCTIONS

In Lesson 10.2.2, students combine functions using addition and subtraction. This is done in the context of different situations.

Example 1

Thomas has $750 saved and earns $228 each day at work, all of which goes into his bank account. Thomas spends $133 each week.

Write one equation \( B(x) \) for the amount of money Thomas has in his bank account after \( x \) weeks. Write another equation \( S(x) \) for the amount of money Thomas has spent after \( x \) weeks. Then combine the equations to create a function for the amount of money Thomas has left after \( x \) weeks.

Solution: \( B(x) = 228x + 750 \) \( S(x) = 133x \)

Since Thomas is saving money in the bank, but then spending it, we need to subtract the amount he spends from the amount he has in the bank.

\[ B(x) – S(x) = (228x + 750) – (133x) \text{ or } B(x) – S(x) = 95x + 750 \]

Example 2

Javier and Earnest are raking leaves for a neighbor with a large yard and lots of trees. Javier can rake 2.5 bags of leaves per hour and at noon has raked 7 bags of leaves. Earnest can rake 3 bags of leaves per hour, but he arrived late this morning and only has 2 bags of leaves raked at noon.

Write one equation \( J(x) \) for the number of bags of leaves Javier has raked \( x \) hours after noon. Write another equation \( E(x) \) for the number of bags of leaves Earnest has raked \( x \) hours after noon. Then combine the equations to create a function for the total number of bags of leaves they have raked together \( x \) hours after noon.

Solution: \( J(x) = 2.5x + 7 \) \( E(x) = 3x + 2 \)

Since Javier and Earnest are working together, we need to add their equations to write a function for the total number of bags of leaves rakes.

\[ J(x) + E(x) = (2.5x + 7) + (3x + 2) \text{ or } J(x) + E(x) = 5.5x + 9 \]
Problems

1. Rex owns a business. It currently costs him $2000 per month to operate his business, but the operational costs are increasing by $10 each month. Rex’s business currently makes a revenue of $3000 per month and the revenue is increasing by $35 each month.

Write one equation for the amount it costs Rex to run his business, another equation for the revenue that his business earns, and a combined equation to model the profit Rex’s business makes after $m$ months.

2. Randy owns two plots of land on which he grows and sells trees. In Plot A there are 982 trees and Randy sells 5 of these trees each day. In Plot B there are 35 trees and Randy plants 4 trees per day.

If $d =$ the number of days that have passed, write one equation for the number of trees in Plot A, another equation for the number of trees in Plot B, and a combined equation for the total number of trees on Randy’s two plots.

For problems 3 through 8, write an equation for the combined functions given the functions below.

\[ f(x) = -4x - 7 \]
\[ g(x) = 3^x + 6 \]
\[ h(x) = -7x + 6 \]

3. \[ g(x) - h(x) \]
4. \[ f(x) + g(x) \]
5. \[ g(x) + h(x) \]
6. \[ h(x) - g(x) \]
7. \[ f(x) - g(x) \]
8. \[ f(x) + h(x) \]

Answers

1. \[ C(m) = 10m + 2000 \quad R(m) = 35m + 3000 \] 
   \[ P(m) = R(m) - C(m) = (35m + 3000) - (10m + 2000) = 25m + 1000 \]
2. \[ A(d) = -5d + 982 \quad B(d) = 4d + 35 \] 
   \[ T(d) = A(d) + B(d) = (-5d + 982) + (4d + 35) = -1d + 1017 \]
3. \[ g(x) - h(x) = (3^x + 6) - (-7x + 6) = 3^x + 7x \]
4. \[ f(x) + g(x) = (-4x - 7) + (3^x + 6) = 3^x - 4x - 1 \]
5. \[ g(x) + h(x) = (3^x + 6) + (-7x + 6) = 3^x - 7x + 12 \]
6. \[ h(x) - g(x) = (-7x + 6) - (3^x + 6) = -7x - 3^x \]
7. \[ f(x) - g(x) = (-4x - 7) - (3^x + 6) = -3^x - 4x - 13 \]
8. \[ f(x) + h(x) = (-4x - 7) + (-7x + 6) = -11x - 1 \]