WORK AND MIXTURE PROBLEMS

Work problems are solved using the concept that if a job can be completed in \(x\) units of time, then the rate (or fraction of the job completed per unit of time) is \(\frac{1}{x}\). For example, if Diann can complete a job in 5 hours, then her rate is \(\frac{1}{5}\) of the job per hour.

Mixture problems are solved using the concept that the product (value or \%) (quantity) must be consistent throughout the equation.

Example 1 (Work)

John can completely wash and dry the dishes in 20 minutes. His brother can do it in 30 minutes. How long will it take them working together?

Solution: John works at a rate of \(\frac{1}{20}\) of the job per minute. His brother works at a rate of \(\frac{1}{30}\) of the job per minute. Since they are working together we add the two rates together to get the combined rate.

Let \(t\) = the time (minutes) to complete the task and then using dimensional analysis:

\[
\left( \frac{1 \text{ job}}{20 \text{ minutes}} + \frac{1 \text{ job}}{30 \text{ minutes}} \right) (t \text{ minutes}) = 1 \text{ job}
\]

Solving the equation:

\[
\left( \frac{5 \text{ job}}{60 \text{ minutes}} \right) (t \text{ minutes}) = 1 \text{ job}
\]

\[
\frac{5}{60} t = 1 \\
\frac{5}{5} t = 60 \text{ minutes} = 12 \text{ minutes}
\]

Example 2 (Work)

With two inflowing pipes open, a water tank can be filled in 5 hours. If the larger pipe can fill the tank alone in 7 hours, how long would the smaller pipe take to fill the tank?

Solution: In this problem, we know the rate of the large pipe \(\left( \frac{1 \text{ job}}{7 \text{ hours}} \right)\), but we do not know the rate of the small pipe. We do know the time. Using the same set-up as Example 1, let \(t\) = time (hours) and the equation is:

\[
\left( \frac{1 \text{ job}}{7 \text{ hours}} + \frac{1 \text{ job}}{t \text{ hours}} \right) (5 \text{ hours}) = 1 \text{ job}
\]

Solving the equation:

\[
\frac{5}{7} + \frac{5}{t} = 1 \\
7t \left( \frac{5}{7} + \frac{5}{t} \right) = 1(7t) \\
5t + 35 = 7t \\
35 = 2t \\
17.5 \text{ hours} = t
\]
**Example 3 (Mixture)**

Alicia has 10 liters of an 80% acid solution. How many liters of water should she add to form a 30% acid solution?

Solution: Draw a diagram to understand the problem.

If the dark gray represents the acid, the amount of acid (in liters) from the two containers on the right must equal the amount of acid in the one container on the left. Using this fact and the concept that (value or percentage) x (quantity) = amount of acid, write and solve the following equation:

\[
(0.80)(10) + (0)(x) = (0.30)(10 + x)
\]

\[
8 = 3 + 0.3x
\]

\[
5 = 0.3x
\]

\[
x = \frac{5}{0.3} = \frac{50}{3} \text{ or } 16 \frac{2}{3} \text{ L}
\]

**Example 4 (Mixture)**

A store has candy worth $0.90 a pound and candy worth $1.20 a pound. If the owners want 60 pounds of candy worth $1.00 a pound, how many pounds of each candy should they use?

Solution: Draw a diagram to understand the problem.

In this problem there are two unknowns, but we can still use the idea of Example 3 to solve this problem. Since there are two unknowns, we will need to write two equations (a system of equations). The cost of the two “bags” of candy on the right must equal the cost of the combined amount on the left. Using this fact and the concept that (value or price) x (quantity) = cost write and solve the following system of equations:

\[
x + y = 60 \quad \text{rewrite as } y = 60 - x \quad \Rightarrow \quad 0.9x + 1.2(60 - x) = 60
\]

\[
0.90x + 1.20y = 1.00(60) \quad \text{then use substitution} \quad 0.9x + 72 - 1.2x = 60
\]

\[
-0.3x = -12
\]

\[
x = 40 \text{ lbs}
\]

Therefore \( y = 20 \text{ lbs} \)

Note: This problem could have been solved with one variable. If the first bag has \( x \) lbs of candy and the final bag has 60 lbs of candy, then the second bag would have \( (60 - x) \) lbs of candy. In this case, start with the equation on the top right.
Problems

1. Susan can paint her living room in 2 hours. Her friend Jaime estimates it would take him 3 hours to paint the same room. If they work together, how long will it take them to paint Susan’s living room?

2. Professor Minh can complete a set of experiments in 4 hours. Her assistant can do it in 6 hours. How long will it take them to complete the experiments working together?

3. With one hose a swimming pool can be filled in 12 hours. Another hose can fill it in 16 hours. How long will it take to fill the pool using both hoses?

4. Together, two machines can harvest a tomato crop in 6 hours. The larger machine can do it alone in 10 hours. How long does it take the smaller machine to harvest the crop working alone?

5. Steven can look up 20 words in a dictionary in an hour. His teammate Mary Lou can look up 30 words per hour. Working together, how long will it take them to look up 100 words?

6. A water tank is filled by one pump in 6 hours and is emptied by another pump in 12 hours. If both pumps are operating, how long will it take to fill the tank?

7. Two crews can service the space shuttle in 12 days. The faster crew can service the shuttle in 20 days alone. How long would the slower crew need to service the shuttle working alone?

8. Janelle and her assistant Ryan can carpet a house in 8 hours. If Janelle could complete the job alone in 12 hours, how long would it take Ryan to carpet the house working alone?

9. Able can harvest a strawberry crop in 4 days. Barney can do it in 5 days. Charlie would take 6 days. If they all work together, how long will it take them to complete the harvest?

10. How much coffee costing $6 a pound should be mixed with 3 pounds of coffee costing $4 a pound to create a mixture costing $4.75 a pound?

11. Sam’s favorite recipe for fruit punch requires 12% apple juice. How much pure apple juice should he add to 2 gallons of punch that already contains 8% apple juice to meet his standards?

12. Jane has 60 liters of 70% acid solution. How many liters of water must be added to form a solution that is 40% acid?

13. How many pound of nuts worth $1.05 a pound must be mixed with nuts worth $0.85 a pound to get a mixture of 200 pounds of nuts worth $0.90 a pound?
14. A coffee shop mixes Kona coffee worth $8 per pound with Brazilian coffee worth $5 per pound. If 30 pounds of the mixture is to be sold for $7 per pound, how many pounds of each coffee should be used?

15. How much tea costing $8 per pound should be mixed with 2 pounds of tea costing $5 per pound to get a mixture costing $6 per pound?

16. How many liters of water must evaporate from 50 liters of an 8% salt solution to make a 25% salt solution?

17. How many gallons of pure lemon juice should be mixed with 4 gallons of 25% lemon juice to achieve a mixture which contains 40% lemon juice?

18. Brian has 20 ounces of a 15% alcohol solution. How many ounces of a 50% alcohol solution must he add to make a 25% alcohol solution?

**Answers**

1. \( \left( \frac{1}{2} + \frac{1}{3} \right) t = 1 \)  
   \( 1.2 \) hours

2. \( \left( \frac{1}{4} + \frac{1}{6} \right) t = 1 \)  
   2.4 hours

3. \( \left( \frac{1}{12} + \frac{1}{16} \right) t = 1 \)  
   \( 6 \frac{5}{16} \approx 6.86 \) hours

4. \( \left( \frac{1}{10} + \frac{1}{5} \right) (6) = 1 \)  
   15 hours

5. \( (20 + 30) t = 100 \)  
   2 hours

6. \( \left( \frac{1}{6} - \frac{1}{12} \right) t = 1 \)  
   12 hours

7. \( \left( \frac{1}{20} + \frac{1}{5} \right) (12) = 1 \)  
   30 days

8. \( \left( \frac{1}{12} + \frac{1}{3} \right) (8) = 1 \)  
   24 hours

9. \( \left( \frac{1}{4} + \frac{1}{5} + \frac{1}{6} \right) t = 1 \)  
   \( \frac{60}{37} \approx 1.62 \) days

10. \( 6x + 4(3) = 4.75(x + 3) \)  
    1.8 pounds

11. \( \frac{1}{11} + x = 0.08(2 + x) \)  
    \( \frac{1}{11} \) gallon

12. \( 0.7(60) + 0.4(60 + x) = 45 \)  
    45 liters

13. \( 1.05x + 0.85y = 0.90(200) \)  
    \( x + y = 200 \)  
    50 pounds

14. \( 8k + 5b = 7(30) \)  
    \( k + b = 30 \)  
    20 Kona, 10 Brazilian

15. \( 8x + 5(2) = 6(x + 2) \)  
    1 pound

16. \( 0.08(50) - 0.25(50 - x) = 34 \)  
    34 liters

17. \( 1x + 0.25(4) = 0.40(x + 4) \)  
    1 gallon

18. \( 0.15(20) + 0.50x = 0.25(20 + x) \)  
    8 ounces