Laws of Exponents

In general, to simplify an expression that contains exponents means to eliminate parentheses and negative exponents if possible. The basic laws of exponents are listed below.

1. \(x^a \cdot x^b = x^{a+b}\)  
   Examples: \(x^3 \cdot x^4 = x^{3+4} = x^7\)  
   \(2^7 \cdot 2^4 = 2^{7+4} = 2^{11}\)

2. \(\frac{x^a}{x^b} = x^{a-b}\)  
   Examples: \(\frac{x^{10}}{x^4} = x^{10-4} = x^6\)  
   \(\frac{2^4}{2^7} = 2^{4-7} = 2^{-3}\) or \(\frac{1}{2^3}\)

3. \((x^a)^b = x^{ab}\)  
   Examples: \((x^4)^3 = x^{4 \cdot 3} = x^{12}\)  
   \((2x^3)^5 = 2^{1 \cdot 5} \cdot x^{3 \cdot 5} = 2^5 \cdot x^{15} = 32x^{15}\)

4. \(x^0 = 1\)  
   Examples: \(2^0 = 1\)  
   \((-3)^0 = 1\)  
   \((\frac{1}{4})^0 = 1\)

5. \(x^{-n} = \frac{1}{x^n}\)  
   Examples: \(x^{-3} = \frac{1}{x^3}\)  
   \(y^{-4} = \frac{1}{y^4}\)  
   \(\frac{1}{4^{-2}} = \frac{1}{\frac{1}{4^2}} = 16\)

In all expressions with fractions we assume the denominator does not equal zero.

For additional information, see the Math Notes box in Lesson 1.3.2. For additional examples and practice, see the Checkpoint 4 materials.

Example 1
\[(2xy^3)(5x^2y^4)\]
Reorder: \(2 \cdot 5 \cdot x \cdot x^2 \cdot y^3 \cdot y^4\)
Using law (1): \(10x^3y^7\)

Example 2
\[\frac{14x^2y^{12}}{7x^5y^7}\]
Separate: \(\frac{14}{7} \cdot \frac{x^2}{x^5} \cdot \frac{y^{12}}{y^7}\)
Using laws (2) and (5): \(2x^{-3}y^5 = \frac{2y^5}{x^3}\)
Example 3
\[(3x^2y^4)^3\]
Using law (3): \[3^3 \cdot (x^2)^3 \cdot (y^4)^3\]
Using law (3) again: \[27x^6y^{12}\]

Example 4
\[(2x^3)^{-2}\]
Using law (5): \[\frac{1}{(2x^3)^2}\]
Using law (3): \[\frac{1}{2^2 \cdot (x^3)^2}\]
Using law (3) again: \[\frac{1}{4x^6}\]

Example 5
Simplify: \[\frac{10x^7y^3}{15x^{-2}y^3}\]
Separate: \[\left(\frac{10}{15}\right) \left(\frac{x^7}{x^{-2}}\right) \left(\frac{y^3}{y^3}\right)\]
Using law (2): \[\frac{2}{3} x^9 y^0\]
Using law (4): \[\frac{2}{3} x^9 \cdot 1 = \frac{2}{3} x^9 = \frac{2x^9}{3}\]

Problems

Simplify each expression. Final answers should contain no parentheses or negative exponents.

1. \(y^5 \cdot y^7\)
2. \(b^4 \cdot b^3 \cdot b^2\)
3. \(8^6 \cdot 8^{-2}\)
4. \((y^5)^2\)
5. \((3a)^4\)
6. \(\frac{m^8}{m^3}\)
7. \(\frac{12m^8}{6m^3}\)
8. \((x^3y^2)^3\)
9. \(\frac{(y^4)^2}{(y^3)^2}\)
10. \(\frac{15x^2y^5}{3x^4y^5}\)
11. \((4c^4)(ac^3)(3a^6c)\)
12. \((7x^3y^5)^2\)
13. \((4xy^2)(2y)^3\)
14. \(\left(\frac{4}{x^2}\right)^3\)
15. \(\frac{(2a^7)(3a^2)}{6a^3}\)
16. \(\left(\frac{5m^3n}{m^5}\right)^3\)
17. \((3a^2x^3)^2(2ax^4)^3\)
18. \(\left(\frac{x^3y}{y^4}\right)^4\)
19. \(\left(\frac{6x^8y^2}{12x^6y^7}\right)^2\)
20. \(\frac{(2x^4y^3)^2(4xy^4)^2}{8x^7y^{12}}\)
21. \(x^{-3}\)
22. \(2x^{-3}\)
23. \((2x)^{-3}\)
24. \((2x^3)^0\)
25. \(5^{-2} \cdot 3\)
26. \(\left(\frac{2x}{3}\right)^{-2}\)
**Scientific Notation**

Scientific notation is a way of writing very large and very small numbers compactly. A number is said to be in scientific notation when it is written as the product of two factors as described below.

- The first factor is less than 10 and greater than or equal to 1.
- The second factor has a base of 10 and an integer exponent.
- The factors are separated by a multiplication sign.
- A positive exponent indicates a number whose absolute value is greater than 1.
- A negative exponent indicates a number whose absolute value is less than 1.

<table>
<thead>
<tr>
<th>Scientific Notation</th>
<th>Standard Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.32 \times 10^{11}$</td>
<td>532,000,000,000</td>
</tr>
<tr>
<td>$2.61 \times 10^{-15}$</td>
<td>0.0000000000000261</td>
</tr>
</tbody>
</table>

**Answers**

1. $y^{12}$  
   2. $b^9$  
   3. $8^4$  
   4. $y^{10}$  
   5. $81a^4$  
   6. $m^5$  
   7. $2m^{11}$  
   8. $x^9y^6$  
   9. $y^2$  
   10. $\frac{5}{x^2}$  
   11. $12a^6c^8$  
   12. $49x^6y^{10}$  
   13. $32xy^5$  
   14. $\frac{64}{x^6}$  
   15. $a^6$  
   16. $\frac{125n^3}{m^6}$  
   17. $72a^7x^{18}$  
   18. $\frac{x^{12}}{y^{12}}$  
   19. $\frac{10}{4y^{10}}$  
   20. $16x^{10}y^5$  
   21. $\frac{1}{x^3}$  
   22. $\frac{2}{x^5}$  
   23. $\frac{1}{8x^5}$  
   24. $1$  
   25. $3$  
   26. $\frac{9}{4x^2}$
It is important to note that the exponent does not necessarily mean to use that number of zeros.

The number $5.32 \times 10^{11}$ means $5.32 \times 100,000,000,000$. Thus, two of the eleven decimal places in the standard form of the number are the 3 and the 2 in 5.32. Standard form in this case is 532,000,000,000. In this example you are moving the decimal point eleven places to the right to write the standard form of the number.

The number $2.61 \times 10^{-15}$ means $2.61 \times 0.000000000000001$. You are moving the decimal point to the left 15 places to write the standard form. Here the standard form is 0.00000000000000261.

For additional information, see the Math Notes box in Lesson 1.3.1.

**Example 1**

Write each number in standard form.

$7.84 \times 10^8 \Rightarrow 784,000,000 \quad \text{and} \quad 3.72 \times 10^{-3} \Rightarrow 0.00372$

When taking a number in standard form and writing it in scientific notation, remember there is only *one* digit to the left of the decimal point allowed.

**Example 2**

Write each number in scientific notation.

$52,050,000 \Rightarrow 5.205 \times 10^7 \quad \text{and} \quad 0.000372 \Rightarrow 3.72 \times 10^{-4}$

The exponent denotes the number of places you moved the decimal point in the standard form. In the first example above, the decimal point is at the end of the number and it was moved 7 places. In the second example above, the exponent is negative because the original number is very small, that is, less than 1.
Problems

Write each number in standard form.

1. $7.85 \times 10^{11}$  2. $1.235 \times 10^9$  3. $1.2305 \times 10^3$  4. $3.89 \times 10^{-7}$  5. $5.28 \times 10^{-4}$

Write each number in scientific notation.

6. $391,000,000,000$  7. $0.0000842$  8. $123056.7$  9. $0.000000502$

10. $25.7$  11. $0.035$  12. $5,600,000$  13. $1346.8$

14. $0.000000000006$  15. $634,700,000,000,000$

Note: On your scientific calculator, displays like $4.357 \times 10^{12}$ (or $4.357 \mathrm{E} 12$) and $3.65 \times 10^{-3}$ (or $3.65 \mathrm{E} -3$) are numbers expressed in scientific notation. The first number means $4.357 \times 10^{12}$ and the second means $3.65 \times 10^{-3}$. The calculator does this because there is not enough room on its display window to show the entire number.

Answers

1. $785,000,000,000$  2. $1,235,000,000$  3. $1230.5$

4. $0.000000389$  5. $0.000528$  6. $3.91 \times 10^{11}$

7. $8.42 \times 10^{-5}$  8. $1.230567 \times 10^5$  9. $5.02 \times 10^{-7}$

10. $2.57 \times 10^1$  11. $3.5 \times 10^{-2}$  12. $5.6 \times 10^6$

13. $1.3468 \times 10^3$  14. $6.0 \times 10^{-12}$  15. $6.347 \times 10^{14}$