### Dimensional Analysis 2.2.4

Dimensional analysis (or unit analysis) means using the rules for multiplying and simplifying fractions to solve problems involving different units.

Unit conversion equations are written as a fraction (equivalent to multiplying by “one”) so that the unwanted units are removed during simplification and the desired units remain.

For information about conversion factors, metric prefixes, and common abbreviations, see the Math Notes box in Lesson 2.2.4. For additional information about dimensional analysis, see the Math Notes box in Lesson 2.3.1.

#### Example 1

A driveway is \(15 \frac{3}{4}\) feet long. How long is this in inches?

Use: \(12 \text{ inches} = 1 \text{ foot}\)

We want the “foot” units to cancel so the fraction we want is: \(\frac{12 \text{ inches}}{1 \text{ foot}}\).

\[
15 \frac{3}{4} \text{ feet} = \frac{63 \text{ feet}}{4} \cdot \frac{12 \text{ inches}}{1 \text{ foot}} = \frac{756 \text{ inches}}{4} = 189 \text{ inches}
\]

#### Example 2

An automobile web site on the Internet advertises that the new *Neptune Stratus* averages \(15\) kilometers per liter of petroleum. What is the equivalent in miles per gallon?

Use: \(1 \text{ gallon} = 3.79 \text{ liters} \quad 1 \text{ mile} = 1.61 \text{ kilometers}\)

We want the “kilometer” and “liter” units to cancel. Use the fractions: \(\frac{1 \text{ mile}}{1.61 \text{ kilometers}}\) and \(\frac{3.79 \text{ liters}}{1 \text{ gallon}}\).

\[
15 \text{ kilometers/liter} = 15 \frac{\text{ kilometers}}{\text{ liter}} \cdot \frac{3.79 \text{ liters}}{1 \text{ gallon}} \cdot \frac{1 \text{ mile}}{1.61 \text{ kilometers}} = \frac{15 \times 3.79 \text{ liters} \cdot \text{ mile}}{1 \text{ gallon} \cdot 1.61 \text{ kilometers}} = 35.3 \text{ miles/gallon}
\]

#### Example 3

A container is strong enough not to break under a weight of \(40\) pounds per square inch \(\left(\frac{40 \text{ pounds}}{(\text{inch})^2}\right)\).

What is the equivalent in grams per square centimeter \(\left(\frac{\text{grams}}{(\text{centimeter})^2}\right)\)?

Use: \(1 \text{ kilogram} = 1000 \text{ grams} = 2.2 \text{ pounds} \quad 1 \text{ inch} = 2.54 \text{ centimeters}\)

We want “pounds” and “inches” unit to cancel. Use the fractions: \(\frac{1000 \text{ grams}}{2.2 \text{ pounds}}\) and \(\frac{1 \text{ inch}}{2.54 \text{ centimeter}}\).

Notice that to cancel the \((\text{inch})^2\) we will need to multiply by the second fraction twice.

\[
\frac{40 \text{ pounds}}{(\text{inch})^2} = \frac{40 \text{ pounds}}{(\text{inch})^2} \cdot \frac{1000 \text{ grams}}{2.2 \text{ pounds}} \cdot \frac{1 \text{ inch}}{2.54 \text{ cm}} \cdot \frac{1 \text{ inch}}{2.54 \text{ cm}} = \frac{40 \times 1000 \text{ grams} \cdot 1 \cdot 1}{2.2 \times 2.54 \times 2.54} \approx 2800 \frac{\text{grams}}{(\text{centimeter})^2}
\]
Use unit analysis and the following conversion equations to solve each problem.

<table>
<thead>
<tr>
<th>1 hour = 60 minutes</th>
<th>1 week = 7 days</th>
<th>1 day = 24 hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year = 365 days</td>
<td>1 liter = 1000 milliliters</td>
<td>1 liter = 1000 centimeter³</td>
</tr>
<tr>
<td>1 milliliter = 20 drops</td>
<td>1 kilometer = 0.625 miles</td>
<td>1 mile = 5280 feet</td>
</tr>
<tr>
<td>1 gallon = 3.79 liters</td>
<td>1 inch = 2.54 centimeters</td>
<td>1 gallon = 128 fluid ounces</td>
</tr>
<tr>
<td>1 gallon = 4 quarts</td>
<td>1 meter = 100 centimeters</td>
<td>1 foot = 12 inches</td>
</tr>
<tr>
<td>1 yard = 3 feet</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Problems

1. A 10-kilometer race is how many miles?
2. One year is how many hours?
3. The distance to the moon is about 250,000 miles. How many feet is this?
4. Traveling 50 miles per hour is the same as how many feet per second?
5. Two hundred fluid ounces is how many gallons?
6. Five hundred feet is the same as how many meters?
7. If tile costs $8.50 per square foot, how much does it cost per square inch?
8. If carpet is advertised at $20 per square yard, what is the cost per square foot?
9. A soda can holds 355 milliliters. How many drops is this? How many gallons is this?
10. A worm moved 8 inches in 5 seconds. How many miles per hour is this?
11. A chemistry experiment calls for 2 drops of acid for 100 milliliters of solution. How much acid should be used for one gallon of solution?
12. If I buy $30 of gasoline at $2.75 per gallon and my car gets 34.2 miles per gallon, how far will I travel on my $30?
13. A swimming pool holds 10,000 gallons of water. How many cubic meters is this?
14. John’s go-cart went 10 laps around a 1320-foot track in 12 minutes. How fast was he traveling in kilometers per hour?
15. In October 2007, one euro was worth $1.42. The price of gasoline in Switzerland was 0.85 euros per liter. What was the cost in dollars per gallon?
**Answers**

1. 6.25 miles  
2. 8760 hours  
3. 1,320,000,000 feet  
4. \( \approx 73.3 \frac{\text{feet}}{\text{second}} \)  
5. 1.5625 gallons  
6. \( \approx 152.4 \text{ meters} \)  
7. \( \approx $0.06 \text{ per sq. inch} \)  
8. $2.22 per ft\(^2\)  
9. 7100 drops \( \approx 0.09 \text{ gallons} \)  
10. \( \approx 0.09 \frac{\text{miles}}{\text{hour}} \)  
11. 75.8 drops \( \approx 4 \text{ ml} \)  
12. 373 miles  
13. 3.79 meters\(^3\)  
14. 20 \( \frac{\text{kilometers}}{\text{hour}} \)  
15. \( \approx $4.57 \text{ per gallon} \)