Now that students are familiar with many of the properties of various triangles, quadrilaterals, and special quadrilaterals, they can apply their algebra skills and knowledge of the coordinate grid to study coordinate geometry. In this section, polygons are plotted on coordinate axes. Using familiar ideas, such as the Pythagorean Theorem and slope, students can prove whether or not quadrilaterals have special properties.

See the Math Notes boxes in Lessons 7.2.2 and 7.2.3.

Example 1

On a set of coordinate axes, plot the points $A(-3, -1), B(1, -4), C(5, -1)$, and $D(1, 2)$ and connect them in the order given. Is this quadrilateral a rhombus? Justify your answer.

To show that this quadrilateral is a rhombus, we must show that all four sides are the same length (definition of a rhombus). When we want to determine the length of a segment on the coordinate graph, we use the Pythagorean Theorem. To begin, plot the points on a graph.

Although the shape appears to be a parallelogram, and possibly a rhombus, we cannot base our decision on appearances. To use the Pythagorean Theorem, we outline a slope triangle, creating a right triangle with $DC$ as the hypotenuse. The lengths of the legs of this right triangle are 3 and 4 units. Using the Pythagorean Theorem,

\[ 3^2 + 4^2 = (DC)^2 \]
\[ 9 + 16 = (DC)^2 \]
\[ 25 = (DC)^2 \]
\[ DC = 5 \]

Similarly, we can draw slope triangles for the other three sides of the quadrilateral and use the Pythagorean Theorem again. In each case, we find the lengths are all 5 units. Therefore, since all four sides have the same length, the polygon is a rhombus.
Example 2

On a set of coordinate axes, plot the points $A(-4, 1), B(1, 3), C(8, -1)$, and $D(4, -3)$, and connect them in the order given. Is this quadrilateral a parallelogram? Justify your answer.

When we plot the points, the quadrilateral appears to be a parallelogram, but we cannot base our decision on appearances. To prove it is a parallelogram, we must show that the opposite sides are parallel. On a graph, we show that lines are parallel by showing that they have the same slope. We can use slope triangles to find the slope of each side.

\[
\text{Slope of } \overline{BC} = \frac{-4}{7} = -\frac{4}{7} \quad \text{Slope of } \overline{AD} = \frac{4}{8} = -\frac{1}{2} \\
\text{Slope of } \overline{BA} = \frac{2}{5} \quad \quad \text{Slope of } \overline{DC} = \frac{2}{4} = \frac{1}{2}
\]

Although the values for the slopes of the opposite sides are close, they are not equal. Therefore this quadrilateral is not a parallelogram.

Example 3

On a set of coordinate axes, plot the points $A(-1, 5), B(6, 1), C(-3, -1)$. Determine the midpoints of $\overline{AC}$ and $\overline{AB}$. Then connect the midpoints to draw a midsegment of the triangle. Show that the midsegment is parallel to, and half the length of $\overline{BC}$.

To determine the midpoints of $\overline{AC}$ and $\overline{AB}$, draw slope triangles.

The midpoint of a segment is found by adding half of the change in $x$ ($\frac{1}{2} \Delta x$) and half the change in $y$ ($\frac{1}{2} \Delta y$) to the coordinates of the leftmost endpoint.

For $\overline{AC}$, $\Delta x = 2$ and $\Delta y = 6$, so $\frac{1}{2} \Delta x = 1$ and $\frac{1}{2} \Delta y = 3$. Point $C(-3, -1)$ is the leftmost point, so the midpoint of $\overline{AC}$ is $(-3 + \frac{1}{2} \Delta x, -1 + \frac{1}{2} \Delta y)$ or $(-3 + 1, -1 + 3) = (-2, 2)$.

Repeat this process to determine that the midpoint of $\overline{AB}$: $\Delta x = 7$ and $\Delta y = 4$, so $\frac{1}{2} \Delta x = 3 \frac{1}{2}$ and $\frac{1}{2} \Delta y = -2$.
Point $A$ is the leftmost point, so the midpoint of $\overline{AB}$ is $(-1 + 3 \frac{1}{2}, 5 + (-2)) = (2 \frac{1}{2}, 3)$.

To show that the midsegment is parallel to $\overline{BC}$, we need to show that they have the same slope. The slope of the midsegment is $\frac{1}{4} = \frac{2}{8}$. The slope of $\overline{BC}$ is also $\frac{2}{3}$.

Use your slope triangles and the Pythagorean Theorem to determine the length of the midsegment and the length of $\overline{BC}$.

Midsegment: $4.5^2 + 1^2 = c^2 \quad 11.25 + 1 = c^2 \quad c = 4.61$

$\overline{BC}$: $9^2 + 2^2 = c^2 \quad 85 = c^2 \quad c = 9.22 \quad 4.61$ is half of 9.22
Problems

1. If \(ABCD\) is a rectangle, and \(A(1, 2), B(5, 2),\) and \(C(5, 5),\) what is the coordinate of \(D?\)

2. If \(P(2, 1)\) and \(Q(6, 1)\) are the endpoints of the base of an isosceles right triangle, what is the \(x\)-coordinate of the third vertex?

3. The three points \(S(-1, -1), A(1, 4),\) and \(M(2, -1)\) are vertices of a parallelogram. What are the coordinates of three possible points for the other vertex?

4. Graph the following lines on the same set of axes. These lines enclose a shape. What is the name of that shape? Justify your answer.

\[
\begin{align*}
y &= \frac{3}{2}x + 7 \\
y &= 0.6x \\
y &= -\frac{10}{6}x - 1 \\
y &= -\frac{5}{3}x + 9
\end{align*}
\]

5. If \(W(-4, -5), X(1, 0), Y(-1, 2),\) and \(Z(-6, -3),\) what shape is \(WXYZ?\) Justify your answer.

6. If \(\overline{DT}\) has endpoints \(D(2, 2)\) and \(T(6, 4),\) what is the equation of the line containing the perpendicular bisector of \(\overline{DT}\)?

Answers

1. \((1, 5)\)

2. \((4, 4)\)

3. \((4, 4), (0, -6),\) or \((-2, 4)\)

4. Since the slopes of opposites side are equal, this is a parallelogram. Additionally, since the slopes of intersecting lines are negative reciprocals of each other, they are perpendicular. This means the angles are all right angles, so the figure is a rectangle.

5. The slopes are: \(WX = 1, XY = -1, YZ = 1,\) and \(ZW = -1.\) This shows that \(WXYZ\) is a rectangle.

6. \(y = -2x + 11\)

The midpoint of \(\overline{DT}\) is \((4, 3).\) The slope of \(\overline{DT}\) is \(\frac{1}{2},\) so the perpendicular slope is \(-2.\) Start with \(3 = -2(4) + b\) and solve for \(b.\)