SIMPLIFYING ALGEBRAIC EXPRESSIONS  A.1.1 and A.1.2

Algebra tiles provide students with the opportunity to “see” abstract algebraic expressions and equations with two variables. Regular use of algebra tiles will help students access abstract concepts through the use of concrete physical representations.

In the figures at right, the dimensions of each tile are shown along its sides, and the area is shown on the tile itself. Algebra tiles are named by their areas. For example, the \( x^2 \)-tile is in the upper left corner; it has an area of \( x^2 \).

In algebraic expressions, combining terms that have the same area to write a simpler expression is called **combining like terms**.

For additional information, refer to the Math Notes box in Lesson A.1.1.

The Lesson A.1.1B Resource Page (available at cpm.org or in the eBook) provides algebra tiles for home use. An algebra tiles eTool is also available at cpm.org or in the eBook.

**Example 1**

Write a simplified algebraic expression for the tile collection below.

**Solution:**

\[
xy + x^2 + x + x + y^2 + 1 + x + y^2 \\
\text{or} \\
x^2 + 2y^2 + xy + 3x + 1
\]

**Example 2**

The expression \( x^2 + xy + y^2 + 3xy + y^2 + 7 \) can be rewritten as \( x^2 + xy + 3xy + y^2 + y^2 + 7 \) and simplified to \( x^2 + 4xy + 2y^2 + 7 \) by combining like terms.

**Example 3**

\[
3x^2 - 4x + 3 + -x^2 + 3x - 7 \\
= 3x^2 - x^2 - 4x + 3x + 3 - 7 \\
= 2x^2 - x - 4
\]
An Expression Mat is a physical representation of an algebraic expression. The upper half of an expression mat is the positive (addition) region and the lower half is the negative (subtraction) region. Positive algebra tiles are shaded and negative tiles are blank. (The illustration to the right reminds you that shaded tiles are positive.) A matching pair of tiles with one tile shaded and the other tile blank represents two opposites—with a value of 0. We refer to them as “zero pairs.” (The Lesson A.1.1A Resource Page has an Expression Mat.)

On an Expression Mat, tiles may be removed or moved in one of two “legal” ways:

1. Flip tiles and move them from the negative region to the positive region. That is, change subtraction to adding the opposite.
2. Remove an equal number of opposite tiles (one shaded and one not shaded) that are within the same region. These pairs of opposite tiles have a value of zero.
3. Group tiles that are alike together. That is, combine like terms.

Example 4

Simplify $3x + 2 - (2x - 3)$.

Create Expression Mat:

Flip tiles in subtraction region to addition region:

Remove zero pairs:

Therefore, $3x + 2 - (2x - 3)$ simplifies to $x + 5$.
Example 5

Simplify $1 - (2y - 3) + y - 2$. Create Expression Mat:

Flip tiles in subtraction region to addition region:

Remove zero pairs:

Therefore, $1 - (2y - 3) + y - 2$ simplifies to $2 + (-y)$ or $2 - y$.

Problems

Simplify each expression by combining like terms. Use algebra tiles if needed.

1. $2x^2 + x + 3 + 4x^2 + 3x + 5$
2. $y^2 + 2y + x^2 + 3y^2 + x^2$
3. $x^2 - 3x + 2 + x^2 + 4x - 7$
4. $y^2 + 2y - 3 - 4y^2 - 2y + 3$
5. $4xy + 3x + 2y - 7 + 6xy + 2x + 7$
6. $x^2 - y^2 + 2x + 3y + x^2 + y^2 + 3y$
7. $(4x^2 + 4x - 1) + (x^2 - x + 7)$
8. $(y^2 + 3xy + x^2) + (2y^2 + 4xy - x^2)$
Write the algebraic expression that corresponds to each Expression Mat, then simplify.

9. \[
\begin{array}{c}
+ \\
\hline
\hline
- \\
\end{array}
\]

10. \[
\begin{array}{c}
+ \\
\hline
\hline
- \\
\end{array}
\]

11. \[
\begin{array}{c}
+ \\
\hline
\hline
- \\
\end{array}
\]

12. \[
\begin{array}{c}
+ \\
\hline
\hline
- \\
\end{array}
\]

13. \[
\begin{array}{c}
+ \\
\hline
\hline
- \\
\end{array}
\]

14. \[
\begin{array}{c}
+ \\
\hline
\hline
- \\
\end{array}
\]

Use algebra tiles and an Expression Mat to simplify each expression.

15. \(3 + 5x - 4 - 7x\)

16. \(-x - 4x - 7\)

17. \(-(-x + 3)\)

18. \(4x - (x + 2)\)

19. \(5x - (-3x + 2)\)

20. \(x - 5 - (2 - x)\)

21. \(1 - 2y - 2y\)

22. \(-3x + 5 + 5x - 1\)

23. \(3 - (y + 5)\)

24. \(-(x + y) + 4x + 2y\)

25. \(3x - 7 - (3x - 7)\)

26. \(-(x + 2y + 3) - 3x + y\)

27. \((7x^2 - 6x - 9) - (9x^2 + 3x - 4)\)

28. \((3x^2 - 8x - 4) - (5x^2 + x + 1)\)
Answers

1. $6x^2 + 4x + 8$
2. $4y^2 + 2y + 2x^2$
3. $2x^2 + x - 5$
4. $-3y^2$
5. $10xy + 5x + 2y$
6. $2x^2 + 2x + 6y$
7. $5x^2 + 3x + 6$
8. $3y^2 + 7xy$
9. $3 + (-2) - 4 - (-3) = 0$
10. $3x + 1 - x - (-1) = 2x + 2$
11. $5 - (-2y) - (3) \text{ or } 5 - (-2y + 3) = 2y + 2$
12. $-4x - x - (-2) = -5x + 2$
13. $-(-2y) - 1 \text{ or } -(-2y + 1) = 2y - 1$
14. $3 + (-2y) - (-y) - (-2) = y + 5$
15. $-2x - 1$
16. $-5x - 7$
17. $x - 3$
18. $3x - 2$
19. $8x - 2$
20. $2x - 7$
21. $-4y + 1$
22. $2x + 4$
23. $-y - 2$
24. $3x + y$
25. $0$
26. $-4x - y - 3$
27. $-2x^2 - 9x - 5$
28. $-2x^2 - 9x - 5$
COMPARING ALGEBRAIC EXPRESSIONS

Expressions on two side-by-side Expression Mats can be compared to determine which expression is greater.

To compare two expressions, represent each expression using algebra tiles on its own Expression Mat. Simplify the expression on each Expression Mat by moving or removing tiles using “legal” moves:

- “Flip” tiles (change them from negative to positive and vice versa) and move them from the negative region to the positive region. That is, change subtraction to adding the opposite.
- Remove an equal number of opposite tiles (one shaded and one not shaded) that are within the same region. That is, remove the zero pairs.
- Group tiles that are alike within the same region together. That is, combine like terms.

Continue to make “legal” moves, in any order, until the expressions cannot be simplified any more.

- Remove tiles that are the same from both Expression Mats if necessary.

Compare the expression on the left with the one on the right to determine which expression is greater. If there are variable tiles remaining after simplifying, you do not have enough information to tell which side is greater—depending on what number the variable tile represents, either expression could be larger than the other.

Example 1

The Expression Comparison Mat at right represents the expressions \(-2x + (-3) + 1 - (-x + 3)\) and \(2 + (-3) - (x - 2)\). Use legal moves to simplify and determine which side is greater.

Solution:

Flip tiles and move them from the negative region to the positive region.

Solution continues on next page →
Remove an equal number of opposite tiles (one shaded and one not shaded) that are within the same region. Also remove the same tiles from both Expression Mats.

Students are also asked to record their steps. Different teachers have different expectations, but here are two possible ways to record the steps. The steps may also be done in a different order.

Recording the steps symbolically:

**Left Expression**

\[ \begin{align*}
\text{flip} & \quad -2x + 1 - 3 - (-x + 3) \\
= & \quad 2 - 3 - (x - 2) \\
\text{flip} & \quad -x - 5 \\
= & \quad 1 - x \\
\end{align*} \]

**Right Expression**

\[ \begin{align*}
? & \quad -2x + 1 - 3 + x - 3 \\
= & \quad 2 - 3 - x + 2 \\
? & \quad -x - 5 \\
= & \quad 1 - x \\
\end{align*} \]

Recording the steps with justifications:

<table>
<thead>
<tr>
<th>Left Expression</th>
<th>Right Expression</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2x + 1 - 3 - (-x + 3))</td>
<td>(2 - 3 - (x - 2))</td>
<td>Starting expressions</td>
</tr>
<tr>
<td>(-2x + 1 - 3 + x - 3)</td>
<td>(2 - 3 - (x - 2))</td>
<td>Flip (-x + 3) from “-” to “+”</td>
</tr>
<tr>
<td>(-x - 5)</td>
<td>(2 - 3 - (x - 2))</td>
<td>Combine like terms</td>
</tr>
<tr>
<td>(-x - 5)</td>
<td>(2 - 3 - x + 2)</td>
<td>Flip (x - 2) from “-” to “+”</td>
</tr>
<tr>
<td>(-5)</td>
<td>(2 - 3 + 2)</td>
<td>Remove (-x) from both sides</td>
</tr>
<tr>
<td>(-5)</td>
<td>(1)</td>
<td>Combine like terms</td>
</tr>
</tbody>
</table>
Example 2

Create the expressions \(x + 1 - (-1 - 2x)\) and \(3 + x - 1 - (x - 4)\) and then use legal moves to simplify and determine which side is greater.

Since we do not know the value of \(x\), it is not possible to determine the greater side.

Recording the steps with justifications:

<table>
<thead>
<tr>
<th>Left Expression</th>
<th>Right Expression</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x + 1 - (-1 - 2x))</td>
<td>(3 + x - 1 - (x - 4))</td>
<td>Starting expressions</td>
</tr>
<tr>
<td>(x + 1 + 1 + 2x)</td>
<td>(3 + x - 1 - x + 4)</td>
<td>Flip from “-” to “+”</td>
</tr>
<tr>
<td>(3x + 2)</td>
<td>(6)</td>
<td>Combine like terms</td>
</tr>
<tr>
<td>(3x)</td>
<td>(4)</td>
<td>Remove 2 from both sides</td>
</tr>
</tbody>
</table>
Problems

Write a set of expressions for problem. Use legal moves to simplify and determine which expression greater. Carefully record your steps.

1. \[ \begin{align*}
\text{Left} &: x^2 - 2x + 3 \\
\text{Right} &: x^2 + x + 2
\end{align*} \]

Which is greater?

2. \[ \begin{align*}
\text{Left} &: x - 2x + 3 \\
\text{Right} &: -x + 3 + x
\end{align*} \]

Which is greater?

3. \[ \begin{align*}
\text{Left} &: x^2 + x^2 + x + x \\
\text{Right} &: x^2 + x^2 + x + x
\end{align*} \]

Which is greater?

4. \[ \begin{align*}
\text{Left} &: x^2 - 2x + 3 \\
\text{Right} &: x^2 - 2x + 3
\end{align*} \]

Which is greater?

In problems 5 through 10, record your steps as you use legal moves to simplify each expression and determine which expression has the greater value.

5. Which is greater: \( 6 - (2x - 4) - 3 \) or \( -x - (1 + x) + 4 \)?

6. Which is greater: \( 3x - (2 - x) + 1 \) or \( -5 + 4x + 3 \)?

7. Which is greater: \( -1 + 6x - 2 + 4y - 2x \) or \( y + 5x - (-2 + x) + 3y - 2 \)?

8. Which is greater: \( x^2 - 2x + 6 - (-3x) \) or \( -(3 - x^2) + 5 + 2x \)?

9. Which is greater: \( x + 2 - (2 - 2x) \) or \( 4 + x - 2 - (x - 4) \)?

10. Which is greater: \( 2x + 4 - x - (2) + x^2 \) or \( 3 + x^2 + 4x - (-3 + 3x) \)?

Answers (Expressions and explanations will vary.)

1. \( -4 > -7 \); left side is greater
2. \( -5 < -1 \); right side is greater
3. \( x > 1 \); not enough information
4. \( 4 > 3 \); left side is greater
5. \( 7 > 3 \); left side is greater
6. \( -1 > -2 \); left side is greater
7. \( -3 < 0 \); right side is greater
8. \( 4 > x \); not enough information
9. \( 3x > 6 \); not enough information
10. \( 0 = 0 \); both sides equal
SOLVING EQUATIONS

A.1.5 – A.1.8

An Equation Mat can be used to represent the process of solving an equation. An Equation Mat is created by putting two Expression Mats side by side—one for each side of the equal sign.

When the process of solving an equation ends with different numbers on each side of the equal sign (for example, $2 = 4$), there is no solution to the problem. When the result is the same expression or number on each side of the equation (for example, $x + 2 = x + 2$) it means that there are infinitely many solutions, or all real numbers are solutions.

See the Math Notes box in Lesson A.1.7 for a list of all the legal moves and their corresponding algebraic language. Also see the Math Notes box in Lesson A.1.8 for solving a linear equation and checking the solution.

For additional examples and practice, see the Checkpoint 1 materials.

Example 1

Solve $x + 2 - (-2x) = x + 5 - (x - 3)$.

First, build the equation on an Equation Mat.

Second, flip the tiles in the subtraction region to the addition region (change subtraction to adding the opposite).

Continue to simplify using legal moves. For example, remove zero pairs.

Example continues on next page →
Example continued from previous page.

Isolate $x$-terms on one side and non-$x$-terms on the other by placing or removing matching tiles from both sides of the Equation Mat. Remove zero pairs again if needed.

Finally, arrange tiles into equal-sized groups on both sides. Since both sides of the equation are equal, determine the value of $x$. In this case, the tiles can be arranged into three groups, resulting in $x = 2$.

Example 2

Solve $3x + 3x - 1 = 4x + 9$

$3x + 3x + (-1) = 4x + 9$  
Flip all tiles from subtraction region to addition region.

$6x + (-1) = 4x + 9$  
Combine like terms.

$6x = 4x + 10$  
Add 1 to each side, remove zero pairs.

$2x = 10$  
Remove 4$x$ from each side.

$x = 5$  
Arrange into two groups.

Example 3

Solve $-2x + 1 - (-3x + 3) = -4 + (-x - 2)$

$-2x + 1 + 3x + (-3) = -4 + (-x) + (-2)$  
Flip all tiles from subtraction region to addition region.

$x + (-2) = (-x) + (-6)$  
Combine like terms.

$x = (-x) + (-4)$  
Add 2 to each side, remove zero pairs.

$2x = -4$  
Add $x$ to both sides, remove zero pairs.

$x = -2$  
Arrange into two groups.
Problems

Solve each equation.

1. \(2x - 3 = -x + 3\)

2. \(1 + 3x - x = x - 4 + 2x\)

3. \(4 - 3x = 2x - 6\)

4. \(3 + 3x - (x - 2) = 3x + 4\)

5. \(-(x + 3) = 2x - 6\)

6. \(-4 + 3x - 1 = 2x + 1 + 2x\)

7. \(-x + 3 = 10\)

8. \(5x - 3 + 2x = x + 7 + 6x\)

9. \(4y - 8 - 2y = 4\)

10. \(9 - (1 - 3y) = 4 + y - (3 - y)\)

11. \(2x - 7 = -x - 1\)

12. \(-2 - 3x = x - 2 - 4x\)

13. \(-3x + 7 = x - 1\)

14. \(1 + 2x - 4 = -3 - (-x)\)

15. \(2x - 1 - 1 = x - 3 - (-5 + x)\)

16. \(-4x - 3 = x - 1 - 5x\)

17. \(10 = x + 6 + 2x\)

18. \(-(x - 2) = x - 5 - 3x\)

19. \(6 - x - 3 = 4x - 8\)

20. \(0.5x - (-x + 3) = x - 5\)

Answers

1. \(x = 2\)

2. \(x = 5\)

3. \(x = 2\)

4. \(x = 1\)

5. \(x = 1\)

6. \(x = -6\)

7. \(x = -7\)

8. no solution

9. \(y = 6\)

10. \(y = -7\)

11. \(x = 2\)

12. all real numbers

13. \(x = 2\)

14. \(x = 0\)

15. \(x = 2\)

16. no solution

17. \(x = 1\frac{1}{3}\)

18. \(x = -7\)

19. \(x = 2\frac{1}{3}\)

20. \(x = -4\)