Students have calculated circumferences and areas of circles and parts of circles, and have used circle properties in applications of probability. This section places the circle on a coordinate graph so that the students can derive the equation of a circle.

For additional information see the Math Notes box in Lesson 10.1.2.

Example 1

What is the equation of the circle centered at the origin with radius 5 units?

The key to deriving the equation of a circle is the Pythagorean Theorem. That means a right triangle within the circle needs to be created. First, draw the circle on graph paper. The coordinates of any point on the circle can be thought of as \((x, y)\). Since the endpoints of the radius to that point are \((0, 0)\) and \((x, y)\), the length of the vertical leg can be represented as \(y\) and the length of the horizontal leg as \(x\). If the radius is represented as \(r\), then using the Pythagorean Theorem we can write \(x^2 + y^2 = r^2\). Since the radius is 5, the equation of this circle can be written as \(x^2 + y^2 = 5^2\), or \(x^2 + y^2 = 25\).

Example 2

Graph the circle \((x - 4)^2 + (y + 2)^2 = 49\).

This is a circle with radius 7 units. This circle, however, is not centered at the origin. The general equation of a circle is \((x - h)^2 + (y - k)^2 = r^2\). The center of the circle is represented by \((h, k)\), so in this example the center is \((4, -2)\).
Example 3

What are the center and the radius of the circle $x^2 - 6x + y^2 + 2y - 5 = 0$?

This equation is not in the graphing form, $(x - h)^2 + (y - k)^2 = r^2$, so it is necessary to “complete the square.” To make a perfect square for $x$, add 9 to both sides of the equation to make a perfect square. For $y$, add 1 to both sides of the equation. (Or add a total of 10 to both sides of the equation). Finally, factor the perfect square trinomials to rewrite the equation in graphing form.

\[
\begin{align*}
    x^2 - 6x + y^2 + 2y - 5 &= 0 \\
    (x^2 - 6x + 9) + (y^2 + 2y + 1) - 5 &= 10 \\
    (x - 3)^2 + (y + 1)^2 &= 15
\end{align*}
\]

The center is $(3, -1)$ and the radius is $\sqrt{15}$ units.

Problems

1. What is the equation of the circle centered at $(0, 0)$ with a radius of 25?
2. What is the equation of the circle centered at the origin with a radius of 7.5?
3. What is the equation of the circle centered at $(5, -3)$ with a radius of 9?

Graph the following circles.

4. $(x + 1)^2 + (y + 5)^2 = 16$
5. $x^2 + (y - 6)^2 = 36$
6. $(x - 3)^2 + y^2 = 64$

Complete the square to rewrite the equation of each circle in graphing form. Identify the center and the radius of each circle.

7. $x^2 + 6x + y^2 - 4y = -9$
8. $x^2 + 10x + y^2 - 8y = -31$
9. $x^2 - 2x + y^2 + 4y - 11 = 0$
10. $x^2 + 9x + y^2 = 0$
Answers

1. \( x^2 + y^2 = 625 \)
2. \( x^2 + y^2 = 56.25 \)
3. \( (x - 5)^2 + (y + 3)^2 = 81 \)

5.

7. \( (x + 3)^2 + (y - 2)^2 = 4; \ (-3, 2), r = 2 \) units
8. \( (x + 5)^2 + (y - 4)^2 = 10; \ (-5, 4), r = \sqrt{10} \approx 3.2 \) units
9. \( (x - 1)^2 + (y + 2)^2 = 16; \ (1, -2), r = 4 \) units
10. \( (x + \frac{9}{2})^2 + y^2 = \frac{81}{4}; \ (-\frac{9}{2}, 0), r = \frac{9}{2} \) units