In this section students develop circle tools, which will help them determine the lengths of chords and measures of angle in circles. As with many topics that students have studied in this course, triangles are useful in solving many of these types of problems.

For additional information see the Math Notes boxes in Lessons 10.2.2, 10.2.3, 10.2.4, and 10.2.5.

Example 1

In the circle at right are two chords, $\overline{AB}$ and $\overline{CD}$. Locate the center of the circle and label it $P$.

The chords of a circle (segments with endpoints on the circle) are useful segments. In particular, the diameter is a special chord that passes through the center. The perpendicular bisector of any chord passes through the center of the circle. Therefore to locate the center, construct the perpendicular bisectors of each chord. The perpendicular bisectors will intersect at the center.

There are several ways to construct perpendicular bisectors of the segments. A quick way is to fold the paper so that the endpoints of the chords come together. The crease will be perpendicular to the chord and bisect it. You can also use a compass-and-straightedge construction. In either case, point $P$ shown in the diagram at right is the center of the circle.
Example 2

In \( \odot O \) at right, use the given information to calculate the values of \( x \), \( y \), and \( z \).

Connected parts of a circle are called arcs, and any pair of points on a circle breaks the circle into two arcs. The larger arc is called a major arc, and the smaller arc is called a minor arc. The length of an arc can be calculated as a fraction of the circumference of its circle. An arc also has a measure based on the measure of its corresponding central angle. In the diagram at right, \( \angle JOE \) is a central angle since its vertex is at the center, \( O \). An arc’s measure is the same as the measure of its central angle. Since \( JE \approx 100^\circ \), \( m \angle JOE = 100^\circ \), or \( x = 100^\circ \).

An angle with its vertex on the circle is called an inscribed angle. Both angles \( y \) and \( z \) are inscribed angles. Inscribed angles measure half of their intercepted arc. In this case \( JE \) is the intercepted arc of both \( \angle y \) and \( \angle z \). Therefore, \( y = z = \frac{1}{2}(100^\circ) = 50^\circ \).

Example 3

In the figure at right, \( O \) is the center of the circle, \( TX \) and \( TB \) are tangent to \( \odot O \), and \( m \angle BOX = 120^\circ \). What is the \( m \angle BTX \)?

A line is tangent to a circle intersects the circle in only one point. Also, a radius drawn to the point of tangency is perpendicular to the tangent line. Therefore, \( OB \perp BT \) and \( OX \perp XT \). One way to solve this problem is to add a segment to the diagram. Adding \( OT \) creates two triangles. These two right triangles are congruent by HL \( \cong \) (\( OB \cong OX \) because they are both radii, and \( OT \cong OT \)). Since the corresponding parts of congruent triangles are also congruent, and \( m \angle BOX = 120^\circ \), we know that \( m \angle BOT = m \angle XOT = 60^\circ \). Since the sum of the angles of a triangle is \( 180^\circ \), \( 60^\circ + 90^\circ + m \angle BTO = 180^\circ \). Therefore \( m \angle BTO = m \angle XTO = 30^\circ \) and \( m \angle BTX = 60^\circ \).

An alternate solution is to note that the two right angles at \( B \) and \( X \), added to \( \angle BOX \), sum to \( 300^\circ \). Since we know that the angles in a quadrilateral sum to \( 360^\circ \), \( m \angle BTX = 360^\circ - 300^\circ = 60^\circ \).
**Example 4**

In the circle at right, \( DV = 9 \) units, \( SV = 12 \) units, and \( AV = 4 \) units. Determine the length of \( IV \).

In the diagram, if we draw \( SI \) and \( DA \) two similar triangles will be formed. (See the Math Notes box in Lesson 10.2.5.) The sides of similar triangles are proportional, so the proportion at right can be written.

Substitute the known lengths, then solve the equation.

\[
\frac{SV}{DV} = \frac{IV}{AV}
\]

\[
\frac{12}{9} = \frac{IV}{4}
\]

\[
9IV = 48
\]

\[
IV \approx 5.3 \text{ units}
\]

**Problems**

Determine each measure in \( \odot P \) if \( m\angle WPX = 28^\circ \), \( m\angle ZPY = 38^\circ \), and \( WZ \) and \( XV \) are diameters.

1. \( m\angleYZ \)  
2. \( m\angleWX \)  
3. \( m\angleVPZ \)  
4. \( m\angleVWX \)

5. \( m\angleXPY \)  
6. \( m\angleXY \)  
7. \( m\angleXWY \)  
8. \( m\angleWZX \)

In each of the following diagrams, \( O \) is the center of the circle. Calculate the value of \( x \) and justify your answer.
In $\odot O$, $m\widehat{WT} = 86^\circ$ and $m\widehat{EA} = 62^\circ$. Calculate:

21. $m\angle EWA$

22. $m\angle WET$

23. $m\angle WES$

24. $m\angle WST$

In $\odot O$, $m\angle EWA = 36^\circ$ and $m\angle WST = 42^\circ$. Calculate:

25. $m\angle WES$

26. $m\angle TW$

27. $m\angle EA$

28. $m\angle TKE$

29. In the diagram at right, $m\angle SD = 92^\circ$, $m\angle DA = 103^\circ$, $m\angle AI = 41^\circ$ and $\overline{SW}$ is tangent to $\odot O$. What are $m\angle AKD$ and $m\angle VAS$?

30. In the diagram at right, $m\angle EK = 43^\circ$, $\overline{EW} \cong \overline{KW}$, and $\overline{ST}$ is tangent to $\odot O$. What are $m\angle WEO$ and $m\angle SEW$?
**Answers**

1. 38°  
2. 28°  
3. 28°  
4. 180°  
5. 114°  
6. 114°  
7. 246°  
8. 332°  
9. 68°  
10. 73°  
11. 98°  
12. 124°  
13. 50°  
14. 55°  
15. 18°  
16. 27°  
17. 55°  
18. 77°  
19. 35°  
20. 50°  
21. $\frac{1}{2}(62°) = 31°$  
22. $\frac{1}{2}(86°) = 43°$  
23. 180° – 43° = 137°  
24. 180° – 137° – 31° = 12°  
25. 180° – 36° – 42° = 102°  
26. $m\angle TEW = 180° − 102° = 78°, 2(78°) = 156°$  
27. 2(36°) = 72°  
28. 180° – 36° − 78° = 66°  
29. $m\angle SAD = \frac{1}{2}(92°), m\angle I DA = \frac{1}{2}(41°), 180° – 46° – 20.5° = 113.5°,$  
$m\angle VAS = 180° – 46° = 134°$  
30. $m\angle EWK = \frac{1}{2}(43°) = 21.5°, m\angle EOK = 43°, so 317° remain for the other angle at O.$  
$m\angle WEO = m\angle WKO$ and for $WEOK, 360° − 21.5° − 317° = 21.5° = m\angle WEO + m\angle WKO,$  
so $m\angle WEO = \frac{1}{2}(21.5°) = 10.75°. m\angle SEO = 90°, m\angle WEO = 10.75°, so m\angle SEW = 79.25°.$