In this chapter, students examine three-dimensional shapes, known as solids. They review how to determine the surface area and volume of prisms and cylinders, including solids that are slanted, or oblique. They also continue to look at similarity, this time studying similar three-dimensional objects, and discover how the linear scale factor can be used to calculate the ratio of the volumes of similar solids.

For additional information see the Math Notes box in Lesson 11.1.3.

Example 1

A cube has an edge length of 20 cm. What are the surface area and volume of this solid?

To calculate the volume, multiply the area of the base by the height. Since the base is a square, its area is 400 square cm. The height is 20 cm, therefore the volume is $(400)(20) = 8000$ cubic cm.

To calculate the surface area, determine the sum of the areas of all six faces. Each face is a square and they are all congruent. The area of one square is 400 square cm, and there are six of them. Therefore the surface area is 2400 square cm.

Example 2

The dimensions of the prism at right are shown. What are the volume and surface area of this prism?

A prism is a special type of solid that has two congruent and parallel bases. The volume of a prism is found by multiplying the area of the base by the height of the prism. To understand this process, think of a prism as a stack of cubes. The base area tells you how many cubes are in one layer of the stack. The height tells you how many layers of cubes are in the solid.

In this example, the base is a right triangle, so the area is $\frac{1}{2} bh$. The top of the prism can be seen as the base.

Base area $= \frac{1}{2} bh = \frac{1}{2} (6)(8) = 24$ sq units, so there are 24 cubes in one layer.
Example continued from previous page.

Calculate the volume by multiplying by the height, 12.

\[ V = (\text{Base area}) \cdot h = (24)(12) = 288 \text{ cubic units} \]

The surface area of this prism is calculated by adding together the areas of the faces, including the bases. One way to illustrate the sub-problems is to make sketches of the surfaces.

\[
\text{Surface Area} = 2 \left( \frac{8}{6} \right) + \frac{12}{6} + \frac{12}{8} + \frac{12}{?} \]

All of the surfaces are familiar shapes, namely, triangles and rectangles. The length of the rectangle on the back face (the last rectangle in the pictorial equation above) is needed. Fortunately, that length is also the hypotenuse of the right triangle of the base, so the Pythagorean Theorem can be used.

\[ 6^2 + 8^2 = ?^2 \]
\[ 36 + 64 = ?^2 \]
\[ ?^2 = 100 \]
\[ ? = \sqrt{100} = 10 \]

Therefore the surface area is:

\[
S.A. = 2 \left( \frac{1}{2} \cdot 6 \cdot 8 \right) + (6 \cdot 12) + (8 \cdot 12) + (10 \cdot 12)
\]
\[ = 48 + 72 + 96 + 120 \]
\[ = 336 \text{ square units} \]

Example 3

The Styrofoam pieces used in packing boxes, known as “shipping peanuts,” are sold in three box sizes: small, medium, and large. The small box has a volume of 1200 cubic inches. The dimensions on the medium box are twice the dimensions of the small box, and the large box has triple the dimensions of the small one. All three boxes are similar prisms. What are the volumes of the medium and large boxes?

Since the boxes are similar, we can use the ratio of similarity to determine the volume of the medium and large boxes without knowing their actual dimensions. When solids are similar with ratio of similarity \( \frac{y}{x} \), the ratio of the areas is \((\frac{y}{x})^2\) and the ratio of the volumes is \((\frac{y}{x})^3\). Since the medium box has dimensions twice the small box and the large box has dimensions three times the small box, we can write:

\[
\frac{\text{medium box}}{\text{small box}} = \frac{2}{1} \quad \Rightarrow \quad \frac{\text{volume of medium box}}{\text{volume of small box}} = \left( \frac{2}{1} \right)^3
\]
\[
\frac{\text{large box}}{\text{small box}} = \frac{3}{1} \quad \Rightarrow \quad \frac{\text{volume of large box}}{\text{volume of small box}} = \left( \frac{3}{1} \right)^3
\]

Solving, \( x = 8 \cdot 1200 \) or volume of the medium box = 9600 cubic inches and \( y = 27 \cdot 1200 \) or volume of large box = 32,400 cubic inches.
Problems

Determine the volume of each solid.

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 

Determine the total surface area of the solids in the previous volume problems.

10. Problem 1  
11. Problem 2  
12. Problem 3  
13. Problem 4  
14. Problem 5  
15. Problem 6  
16. Problem 7  
17. Problem 8  

18. What are the volume and surface area of the solid below?

19. Calculate the volume of the remaining solid after a hole with a diameter of 4 mm is drilled through it.

20. At Cakes R Us, it is possible to buy round cakes in different sizes. The smallest cake has a diameter of 8 inches and a height of 4 inches, and requires 3 cups of batter. Another similar round cake has a diameter of 13 inches. How much batter would this cake require?
21. Two rectangular prisms are similar. The smaller, A, has a height of 4 units while the larger, B, has a height of 6 units.
   
   a. What is the scale factor from prism A to prism B?
   
   b. What is the ratio of the lengths of the edges labeled x and y?
   
   c. What is the ratio of their surface areas?
      What is the ratio of their volumes?
   
   d. A third prism C is similar to prisms A and B. Prism C’s height is 10 units. If the volume of prism A is 24 cubic units, what is the volume of prism C?

22. Prism A and prism B are similar with a ratio of similarity of 2:3. If the volume of prism A is 36 cubic units, what is the volume of prism B?

23. If rectangle A and rectangle B have a ratio of similarity of 5:4, what is the area of rectangle B if the area of rectangle A is 24 square units?

24. Rectangle A is similar to rectangle B. The area of rectangle A is 81 square units while the area of rectangle B is 49 square units. What is the ratio of similarity between the two rectangles?

25. If prism A and prism B have a ratio of similarity of 1:4, what is the volume of prism B if the volume of prism A is 83 cubic units?

26. Prism A and prism B are similar. The volume of prism A is 72 cubic units while the volume of prism B is 1125 cubic units. What is the ratio of similarity between these two prisms?

27. Prism A and prism B are similar. The volume of prism A is 27 cubic units while the volume of prism B is approximately 512 cubic units. If the surface area of prism B is 128 square units, what is the surface area of prism A?

28. The corresponding diagonals of two similar trapezoids are in the ratio of 1:7. What is the ratio of their areas?

29. The ratio of the perimeters of two similar parallelograms is 3:7. What is the ratio of their areas?

30. The areas of two circles are in the ratio of 25:16. What is the ratio of their radii?

31. The ratio of the volumes of two similar circular cylinders is 27:64. What is the ratio of the diameters of their similar bases?

32. The surface areas of two cubes are in the ratio of 49:81. What is the ratio of their volumes?
Answers

1. $48 \text{ m}^3$
2. $540 \text{ cm}^3$
3. $\approx 14967 \text{ ft}^3$
4. $\approx 77.0 \text{ in}^3$
5. $\approx 1508.7 \text{ m}^3$
6. $\approx 157 \text{ m}^3$
7. $72 \text{ ft}^3$
8. $\approx 1045 \text{ cm}^3$
9. $\approx 332.6 \text{ cm}^3$
10. $80 \text{ m}^2$
11. $468 \text{ cm}^2$
12. $\approx 3997 \text{ ft}^2$
13. $\approx 121.0 \text{ in}^2$
14. $\approx 728.3 \text{ m}^2$
15. $\approx 220 \text{ m}^2$
16. $124 \text{ ft}^2$
17. $\approx 570 \text{ cm}^2$
18. $7245 \text{ ft}^3, \approx 2395 \text{ ft}^2$
19. $\approx 1012 \text{ mm}^3$
20. $\approx 13 \text{ cups}$
21. a. $\frac{4}{6} = \frac{2}{3}$
    b. $\frac{x}{y} = \frac{4}{6} = \frac{2}{3}$
    c. $\frac{16}{36} = \frac{4}{9}, \frac{64}{216} = \frac{8}{27}$
    d. $375 \text{ cu. units}$
22. $121.5 \text{ units}^3$
23. $15.36 \text{ units}^2$
24. $\frac{9}{7}$
25. $5312 \text{ units}^3$
26. $\frac{2}{5}$
27. $\approx 18 \text{ units}^2$
28. $\frac{1}{49}$
29. $\frac{9}{49}$
30. $\frac{5}{4}$
31. $\frac{3}{4}$
32. $\frac{343}{729}$