EXPECTED VALUE

In Lesson 3.1.5 students investigate expected value by analyzing different games. Ultimately, students develop a method for calculating expected value. Note that students sometimes think that the expected value must actually be one of the possible outcomes of a game or situation. It does not have to be. Expected value is a calculation of the average expected result for one play if the game is played many times.

See the Math Notes box in Lesson 3.2.1 for more information about expected value.

Example 1

The spinner at right is divided into different sections, each assigned a different point value. The three smaller sections are congruent. If you were to spin the spinner 100 times, how many times would you expect to get each of the different point values? What is the expected value of this spinner?

The angle of each sector is what determines the probability of the spinner landing in that region. Therefore the probability of landing on 6 points is $\frac{1}{2}$ because that region takes up half of the spinner. The other half of the circle is divided into three equal parts, each taking up $\frac{1}{6}$ of the whole spinner ($\frac{1}{3}$ of $\frac{1}{2}$). Now that we know the probabilities, we can determine how many times we would expect each of the values to occur. Since the probability of getting 6 points is $\frac{1}{2}$, we expect that about 50 of the 100 spins (half) will land on 6 points. Similarly, since the probability of landing on 1 point (or 2 or 3 points) is $\frac{1}{6}$, we expect about $\frac{1}{6}$ of the 100 spins to land on each of those, or about 16 or 17 times. If the total number of spins is 100, we can expect on average about 50 of them to be 6 points, $16\frac{2}{3}$ to be 1 point, $16\frac{2}{3}$ to be 2 points, and $16\frac{2}{3}$ to be 3 points. (Note: These are estimates, not exact or guaranteed.) Using these values, after 100 spins, the player would have about $50(6)+16\frac{2}{3}(1)+16\frac{2}{3}(2)+16\frac{2}{3}(3)=400$ points.

It is expected that a player earns 400 points in 100 spins, or an average 4 points per spin. So the expected value is 4 points. Note: 4 points is the expected value for this spinner even though it is NOT one of the possible outcomes.
Example 2

A $3 \times 3$ grid of nine congruent squares is painted various colors. Six of the small squares are painted red while three are painted blue. For $1.00$ a player can throw a dart at the grid. If the player hits a blue square, they win $2.00$. Is this a fair game? Justify your answer.

<table>
<thead>
<tr>
<th>R</th>
<th>R</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>B</td>
<td>B</td>
</tr>
</tbody>
</table>

A fair game is one in which the expected value is $0$ because this means that, on average, the game does not favor the player or the person running the game. To determine if this game is fair we need to calculate its expected value.

The expected value is found by summing the products of the amounts that can be won and their probabilities. In this problem, each game costs $1.00$ to play. If the dart lands on a red square, the player loses $1.00$ (the value is $-1$). The probability of hitting a red square is $\frac{6}{9} = \frac{2}{3}$. However, if the player hits a blue square, the player receives $2.00$, which is a gain of only $1.00$ (because he paid $1.00$ to play). As shown in the calculations at right, the expected value of this game is $-\frac{1}{3}$. Therefore, this is not a fair game; it favors the person running the game.

\[
E = \frac{2}{3}(-1.00) + \frac{1}{3}(1.00) = -\frac{2}{3} + \frac{1}{3} = -\frac{1}{3}
\]
Problems

The spinners below have different point values assigned to different regions. What is the expected value for each spinner? (Assume that regions that appear to be congruent, are congruent.)

1.  
   \[
   \begin{array}{c|c}
   1 & 2 \\
   \hline
   3 & 4 \\
   \end{array}
   \]

2.  
   \[
   \begin{array}{c|c|c}
   5 & 3 \\
   \hline
   2 & 6 \\
   \end{array}
   \]

3.  
   \[
   \begin{array}{c|c|c}
   3 & 2 \\
   \hline
   6 \\
   \end{array}
   \]

4.  
   \[
   \begin{array}{c|c|c}
   1 & 3 \\
   \hline
   5 \\
   \end{array}
   \]

5.  
   \[
   \begin{array}{c|c|c|c}
   2 & 3 & 4 \\
   \hline
   8 \\
   \end{array}
   \]

6.  
   \[
   \begin{array}{c|c|c|c}
   2 & 3 & 1 \\
   \hline
   6 \\
   \end{array}
   \]

7. For $0.40 a player gets one dart to throw at a board that looks like the diagram at right. The board is a square, measuring 1 foot along each side. The circle has a diameter of six inches. For each dart that lands in the circle, the player wins $0.75. Is this game fair? Justify your answer.

Answers

1. 2.5
2. 4
3. \(3 \frac{2}{3}\)
4. 3
5. 5.5
6. 4.75
7. Not fair because the expected value is about $-0.25.