In Lessons 4.1.1 through 4.1.4 students factor quadratic expressions. This prepares them for solving quadratic equations in Chapter 5.

In Chapter 1 students used algebra tiles to build models of quadratic expressions. They then moved from physical tiles to area models, which can more easily represent large numbers of tiles and negative tiles. In the diagram below, the length and width of the rectangle are \((x + 2)\) and \((x + 4)\). Since \((\text{base})(\text{height}) = \text{area}\), the area of the rectangle can be expressed as a product, \((x + 2)(x + 4)\). The small sections of the rectangle also make up its area, so the area can be expressed as a sum, \(4x + 8 + x^2 + 2x\), or \(x^2 + 6x + 8\).

Area as a product = area as a sum, thus students wrote \((x + 2)(x + 4) = x^2 + 6x + 8\).

The length and width of the rectangle, \((x + 2)\) and \((x + 4)\), are factors of the quadratic expression \(x^2 + 6x + 8\), since the product of \((x + 2)\) and \((x + 4)\) is \(x^2 + 6x + 8\). Therefore, the factored form of \(x^2 + 6x + 8\) is \((x + 2)(x + 4)\).

For a more detailed example of the method used by students to factor quadratic expressions, see the Math Notes box in Lesson 4.1.4. For additional information, see the Math Notes boxes in Lessons 4.1.1 and 4.1.2. For additional examples and more practice, see the Checkpoint 7 materials in the student text.

Example 1

Factor \(x^2 + 7x + 12\).

Sketch an area model.

Place the \(x^2\) and the 12 along one diagonal.

Since the products of the diagonals must be the same, identify two terms whose product is \(12x^2\) and whose sum is \(7x\). In this case, \(3x\) and \(4x\). (Students are familiar with this situation as a “diamond problem” from Chapter 1.)

Write these terms along the other diagonal. Either term can go in either diagonal space.

Determine the base and height of the large outer rectangle by using the areas of the small pieces and determining the greatest common factor of each row and each column.

Write the sum as a product (factored form).

\[ x^2 + 7x + 12 = (x + 3)(x + 4) \]
Example 2

Factor \( x^2 + 7x - 30 \).

Sketch an area model.
Place the \( x^2 \) and the \(-30\) along one diagonal.

Since the products of the diagonals must be the same, identify two terms whose product is \(-30x^2\) and whose sum is \(7x\). In this case, \(-3x\) and \(10x\).

Write these terms along the other diagonal. Either term can go in either diagonal space.

Determine the base and height of the large outer rectangle by using the areas of the small pieces and determining the greatest common factor of each row and each column.

Write the sum as a product (factored form).

\[ x^2 + 7x - 30 = (x - 3)(x + 10) \]

Example 3

Factor \( x^2 - 15x + 56 \).

Sketch an area model.
Place the \( x^2 \) and the \(56\) along one diagonal.

Since the products of the diagonals must be the same, identify two terms whose product is \(56x^2\) and whose sum is \(-15x\). Write these terms as the other diagonal.

Determine the base and height of the large outer rectangle by using the areas of the small pieces and determining the greatest common factor of each row and each column.

Write the sum as a product (factored form).

\[ x^2 - 15x + 56 = (x - 7)(x - 8) \]
Example 4

Factor $12x^2 - 19x + 5$.

Sketch an area model.

Place the $12x^2$ and the 5 along one diagonal.

Since the products of the diagonals must be the same, identify two terms whose product is $60x^2$ and whose sum is $-19x$. Write these terms as the other diagonal.

Determine the base and height of the rectangle. Check the signs of the factors.

Write the sum as a product (factored form). $12x^2 - 19x + 5 = (3x - 1)(4x - 5)$

Example 5

Factor $3x^2 + 21x + 36$.

Note: If a common factor appears in all the terms, it should be factored out first.

For example, $3x^2 + 21x + 36 = 3(x^2 + 7x + 12)$.

Then $x^2 + 7x + 12$ can be factored as in Example 1.

$x^2 + 7x + 12 = (x + 3)(x + 4)$.

Then, since the expression $3x^2 + 21x + 36$ has a factor of 3,

$3x^2 + 21x + 36 = 3(x^2 + 7x + 12) = 3(x + 3)(x + 4)$.

Problems

Factor the following expressions.

1. $x^2 + 5x + 6$  
2. $2x^2 + 5x + 3$  
3. $3x^2 + 4x + 1$  
4. $3x^2 + 30x + 75$
5. $x^2 + 15x + 44$  
6. $x^2 + 7x + 6$  
7. $2x^2 + 22x + 48$  
8. $x^2 + 4x - 32$
9. $4x^2 + 12x + 9$  
10. $24x^2 + 22x - 10$  
11. $x^2 + x - 72$  
12. $3x^2 - 20x - 7$
13. $x^3 - 11x^2 + 28x$  
14. $2x^2 + 11x - 6$  
15. $2x^2 + 5x - 3$  
16. $x^2 - 3x - 10$
17. $4x^2 - 12x + 9$  
18. $3x^2 + 2x - 5$  
19. $6x^2 - x - 2$  
20. $9x^2 - 18x + 8$
Answers

1. \((x + 2)(x + 3)\)  
2. \((x + 1)(2x + 3)\)  
3. \((3x + 1)(x + 1)\)  
4. \(3(x + 5)(x + 5)\)  
5. \((x + 11)(x + 4)\)  
6. \((x + 6)(x + 1)\)  
7. \(2(x + 8)(x + 3)\)  
8. \((x + 8)(x - 4)\)  
9. \((2x + 3)(2x + 3)\)  
10. \(2(3x - 1)(4x + 5)\)  
11. \((x - 8)(x + 9)\)  
12. \((x - 7)(3x + 1)\)  
13. \(x(x - 4)(x - 7)\)  
14. \((x + 6)(2x - 1)\)  
15. \((x + 3)(2x - 1)\)  
16. \((x - 5)(x + 2)\)  
17. \((2x - 3)(2x - 3)\)  
18. \((3x + 5)(x - 1)\)  
19. \((2x + 1)(3x - 2)\)  
20. \((3x - 4)(3x - 2)\)