The graph of a quadratic function, a parabola, is a symmetrical curve. Its highest or lowest point is called the vertex. The graph of a parabola can be created by using an equation in the form \( y = ax^2 + bx + c \). In previous lessons students graphed parabolas by substituting values for \( x \) and then evaluating \( y \). This can be a tedious process. Another method for graphing a parabola is to determine the \( x \)-intercepts first, and then solve for the vertex and/or the \( y \)-intercept. To determine the \( x \)-intercepts, substitute 0 for \( y \) and solve the quadratic equation, \( 0 = ax^2 + bx + c \). One method for solving a quadratic equation is to factor and use the Zero Product Property. This method uses two ideas:

1. When a product is equal to zero, then at least one of the factors must be zero.
2. Some quadratic expressions can be factored into the product of two simple binomials.

For additional information see the Math Notes box in Lesson 5.1.4.

### Example 1

Determine the \( x \)-intercepts of the parabola \( y = x^2 + 6x + 8 \). Then find the \( y \)-intercept and sketch the parabola.

First, substitute \( y = 0 \):

\[
0 = x^2 + 6x + 8
\]

Then factor the quadratic expression.

\[
(x + 4)(x + 2) = 0
\]

Set each factor equal to 0.

\[
(x + 4) = 0 \quad \text{or} \quad (x + 2) = 0
\]

Solve each equation for \( x \).

\[
x = -4 \quad \text{or} \quad x = -2
\]

Find the \( y \)-intercept by substituting \( x = 0 \) and evaluating.

\[
y = (0)^2 + 6(0) + 8 = 8
\]

The graph is a parabola with \( x \)-intercepts at \((-4, 0)\) and \((-2, 0)\) and a \( y \)-intercept at \((0, 8)\).
Example 2

Use the \(x\)-intercepts and the vertex to sketch the graph of \(-2x^2 + 7x + 15 = y\).

Substitute \(y = 0\).

\[-2x^2 + 7x + 15 = 0\]

Factor the quadratic expression.

\[(-2x - 3)(x - 5) = 0\]

Set each factor equal to 0.

\[-2x - 3 = 0 \quad \text{or} \quad x - 5 = 0\]

Solve each equation for \(x\).

\[-2x = 3 \quad \text{or} \quad x = 5\]

\[x = -\frac{3}{2} \quad \text{or} \quad x = 5\]

To find the vertex, average the \(x\)-intercepts and use that \(x\)-value to calculate the corresponding \(y\)-value. The average of the \(x\)-intercepts is \(\frac{5 + \frac{-1.5}{2}}{2} = 1.75\). Substituting this into the original equation yields:

\[-2(1.75)^2 + 7(1.75) + 15 = 21.125\]

Thus the vertex is located at \((1.75, 21.125)\). Use the vertex and the \(x\)-intercepts at \((-\frac{3}{2}, 0)\) and \((5, 0)\) to sketch the graph of the parabola.

Example 3

Sketch a graph of \(9x^2 - 6x + 1 = y\).

Substitute \(y = 0\).

\[9x^2 - 6x + 1 = 0\]

Factor the quadratic expression.

\[(3x - 1)(3x - 1) = 0\]

Solve each equation for \(x\). Notice that the factors are the same so there will be only one solution.

\[3x - 1 = 0\]

\[3x = 1\]

\[x = \frac{1}{3}\]

Substitute \(x = 0\) into the original equation to determine that the \(y\)-intercept is \((0, 1)\). The graph is a parabola that has a single \(x\)-intercept at \((\frac{1}{3}, 0)\) and \(y\)-intercept at \((0, 1)\).
Problems

Use the Zero Product Property to determine the \( x \)-intercepts for the graph of each quadratic function.

1. \( y = x^2 + x - 20 \)  
2. \( y = -x^2 - 7x - 12 \)  
3. \( y = 3x^2 - 7x - 6 \)  
4. \( y = 3x^2 + 11x + 10 \)  
5. \( y = 6x^2 + 5x - 4 \)  
6. \( y = -x^2 + 2x + 8 \)  
7. \( y = 6x^2 - x - 15 \)  
8. \( y = 4x^2 + 12x + 9 \)  
9. \( y = 2x^2 + 8x + 6 \)

Determine the \( x \)- and \( y \)-intercepts of the graph of each of the following quadratic functions and then sketch the graph.

10. \( y = x^2 + 4x + 3 \)  
11. \( y = x^2 + 5x - 6 \)  
12. \( y = 2x^2 - 7x - 4 \)  
13. \( y = 2x^2 - 10x + 8 \)  
14. \( y = 16x^2 - 25 \)

Answers

1. \((-5, 0)\) and \((4, 0)\)  
2. \((-4, 0)\) and \((-3, 0)\)  
3. \((-\frac{2}{3}, 0)\) and \((3, 0)\)  
4. \((-\frac{5}{3}, 0)\) and \((-2, 0)\)  
5. \((-\frac{4}{3}, 0)\) and \((\frac{1}{2}, 0)\)  
6. \((4, 0)\) and \((-2, 0)\)  
7. \((-\frac{3}{2}, 0)\) and \((\frac{5}{3}, 0)\)  
8. \((-\frac{3}{2}, 0)\)  
9. \((0, 0)\)  
10. \((0, -6)\)  
11. \((0, -6)\)  
12. \((-1, 0)\) and \((-3, 0)\)  
13. \(x\)-intercepts: \((-1, 0), (-3, 0)\)  
\(y\)-intercept: \((0, 3)\)  
14. \(x\)-intercepts: \((-6, 0), (1, 0)\)  
\(y\)-intercept: \((0, -6)\)  
15. \(x\)-intercepts: \((-0.5, 0), (4, 0)\)  
\(y\)-intercept: \((0, -4)\)  
16. \(x\)-intercepts: \((-4, 0), \left(\frac{2}{3}, 0\right)\)  
\(y\)-intercept: \((0, 8)\)  
17. \(x\)-intercepts: \((-\frac{5}{4}, 0), \left(\frac{5}{4}, 0\right)\)  
\(y\)-intercept: \((0, -25)\)