USING THE QUADRATIC FORMULA

5.2.4

When a quadratic equation is not factorable, another method is needed to solve for \( x \). You can always complete the square to solve a quadratic equation, but that can be challenging when the coefficients are large. The Quadratic Formula can be used to solve any quadratic equation, no matter how complicated.

The solution(s) to any quadratic equation \( ax^2 + bx + c = 0 \) are:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
\]

The \( \pm \) symbol is read as “plus or minus.” It is shorthand notation that tells you to calculate the formula twice, once with + and again with – to get both \( x \)-values.

To use the formula, the quadratic equation must be written in standard form: \( ax^2 + bx + c = 0 \). This is necessary to correctly identify the values of \( a, b, \) and \( c \). Once the equation is in standard form and equal to 0, \( a \) is the coefficient of the \( x^2 \)-term, \( b \) is the coefficient of the \( x \)-term and \( c \) is the constant term.

For additional information, see the Math Notes box in Lesson 5.2.5.

Example 1

Solve \( 2x^2 - 5x - 3 = 0 \).

Identify \( a, b, \) and \( c \). Watch your signs.

\[ a = 2, b = -5, c = -3 \]

Write the Quadratic Formula.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Substitute \( a, b, \) and \( c \) into the formula and do the initial calculations.

\[
x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}
\]

\[ x = \frac{5 \pm \sqrt{25 - (-24)}}{4} \]

Simplify the \( \sqrt{ } \).

\[ x = \frac{5 \pm \sqrt{49}}{4} \]

Calculate both values of \( x \).

\[ x = \frac{5 + 7}{4} = \frac{12}{4} = 3 \quad \text{or} \quad x = \frac{5 - 7}{4} = \frac{-2}{4} = -\frac{1}{2} \]

The solutions are \( x = 3 \) or \( x = -\frac{1}{2} \).
Example 2

Solve $3x^2 + 5x + 1 = 0$.

Identify $a$, $b$, and $c$. \[ a = 3, \ b = 5, \ c = 1 \]

Write the Quadratic Formula.

\[ x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \]

Substitute $a$, $b$, and $c$ into the formula and do the initial calculations.

\[ x = \frac{-5\pm\sqrt{(-5)^2-4(3)(1)}}{2(3)} \]

\[ x = \frac{-5\pm\sqrt{25-12}}{6} \]

Simplify the $\sqrt{}$.

\[ x = \frac{-5\pm\sqrt{13}}{6} \]

The solutions are $x = \frac{-5+\sqrt{13}}{6} \approx -0.23$ or $x = \frac{-5-\sqrt{13}}{6} \approx -1.43$.

Example 3

Solve $25x^2 - 20x + 4 = 0$.

Identify $a$, $b$, and $c$. \[ a = 25, \ b = -20, \ c = 4 \]

Write the Quadratic Formula.

\[ x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \]

Substitute $a$, $b$, and $c$ into the formula and do the initial calculations.

\[ x = \frac{-(-20)\pm\sqrt{(-20)^2-4(25)(4)}}{2(25)} \]

\[ x = \frac{20\pm\sqrt{400-400}}{50} \]

Simplify the $\sqrt{}$.

\[ x = \frac{20\pm\sqrt{0}}{50} \]

This quadratic equation has only one solution: $x = \frac{2}{5}$. 
Example 4

Solve \( x^2 + 4x = -10 \).

Rewrite the equation in standard form.

\[ x^2 + 4x + 10 = 0 \]

Identify \( a, b, \) and \( c \).

\( a = 1, \ b = 4, \ c = 10 \)

Write the Quadratic Formula.

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Substitute \( a, b, \) and \( c \) into the formula and do the initial calculations.

\[ x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(10)}}{2(1)} \]

\[ x = \frac{-4 \pm \sqrt{-24}}{2} \]

There are no real numbers that can be squared to give \(-24\); therefore this quadratic equation has no real solutions.

Problems

Solve each equation by using the Quadratic Formula.

1. \( x^2 - x - 2 = 0 \)
2. \( x^2 - x - 3 = 0 \)
3. \(-3x^2 + 2x + 1 = 0 \)
4. \(-2 - 2x^2 = 4x \)
5. \( 7x = 10 - 2x^2 \)
6. \(-6x^2 - x + 6 = 0 \)
7. \( 6 - 4x + 3x^2 = 8 \)
8. \( 4x^2 + x - 1 = 0 \)
9. \( x^2 - 5x + 3 = 0 \)
10. \( 0 = 10x^2 - 2x + 3 \)
11. \( x(-3x + 5) = 7x - 10 \)
12. \( (5x + 5)(x - 5) = 7x \)

Identify the error in each of the following solutions. Then write a correct solution to the problem.

13. Solve: \( 3x^2 + 6x + 1 = 0 \)

\[ a = 3, \ b = 6, \ c = 1 \]

\[ x = \frac{6 \pm \sqrt{6^2 - 4(3)(1)}}{2(3)} \]

\[ = \frac{6 \pm \sqrt{36 - 12}}{6} \]

\[ = \frac{6 \pm \sqrt{24}}{6} \]

\[ = 1 \pm \frac{\sqrt{24}}{6} \]

14. Solve: \(-2x^2 + 7x + 5 = 0 \)

\[ a = -2, \ b = 7, \ c = 5 \]

\[ x = \frac{-7 \pm \sqrt{7^2 - 4(-2)(5)}}{2(-2)} \]

\[ = \frac{-7 \pm \sqrt{49 - 40}}{-4} \]

\[ = \frac{7 \pm 3}{4} \]

\[ x = \frac{-4}{3} \text{ or } x = \frac{-10}{4} = 2.5 \]
Answers

1. \( x = 2 \) or \(-1\)
2. \( x = \frac{1 + \sqrt{13}}{2} \approx 2.30 \) or \(-1.30\)
3. \( x = -\frac{1}{3} \) or 1

4. \( x = -1 \)
5. \( x = \frac{-7 \pm \sqrt{129}}{4} \approx 1.09 \) or \(-4.59\)
6. \( x = \frac{1 \pm \sqrt{145}}{-12} \approx -1.09 \) or 0.92

7. \( x = \frac{4 \pm \sqrt{40}}{6} = \frac{2 \pm \sqrt{10}}{3} \approx 1.72 \) or \(-0.39\)
8. \( x = -\frac{1 \pm \sqrt{17}}{8} \approx 0.39 \) or \(-0.64\)
9. \( x = \frac{5 \pm \sqrt{13}}{2} \approx 4.30 \) or 0.70

10. no real solution
11. \( x = \frac{2 \pm \sqrt{124}}{-6} = \frac{1 \pm \sqrt{31}}{-3} \approx -2.19 \) or 1.52
12. \( x = \frac{27 \pm \sqrt{1229}}{10} \approx 6.21 \) or \(-0.81\)

13. The formula starts with “\(-b\)”; the negative sign was left off. \( x = -1 \pm \frac{\sqrt{24}}{6} \)

14. Under the radical, “\(-4ac\)” should equal + 40. \( x = \frac{-7 \pm \sqrt{89}}{-4} \)