FRACTIONAL EXPONENTS

6.1.4

A fractional exponent is equivalent to an expression with roots or radicals.

For \( x \neq 0 \) and \( n \neq 0 \), \( x^{m/n} = (x^m)^{1/n} = \sqrt[n]{x^m} \) or \( x^{m/n} = (x^{1/n})^m = (\sqrt[n]{x})^m \).

Fractional exponents can also be used to solve equations containing exponents. For additional information, see the Math Notes box in Lesson 6.2.1.

Example 1

Rewrite each expression in radical form and simplify if possible.

a. \( 16^{5/4} \)

Solution:

\[
\begin{align*}
16^{5/4} &= (16^{1/4})^5 \\
&= (\sqrt[4]{16})^5 \\
&= (2)^5 \\
&= 32
\end{align*}
\]

b. \( (-8)^{2/3} \)

\[
\begin{align*}
(-8)^{2/3} &= ((-8)^{1/3})^2 \\
&= (\sqrt[3]{-8})^2 \\
&= (-2)^2 \\
&= 4
\end{align*}
\]

Example 2

Simplify each expression. Answer should contain no parentheses and no negative exponents.

a. \( (144x^{-12})^{1/2} \)

Using the Laws of Exponents:

\[
\begin{align*}
(144x^{-12})^{1/2} &= \left(\frac{144}{x^{12}}\right)^{1/2} \\
&= \sqrt{144} \cdot \sqrt{\frac{1}{x^{12}}} \\
&= 12 \cdot \frac{1}{x^6} \\
&= \frac{12}{x^6}
\end{align*}
\]
Problems

Rewrite each expression as at least three different equivalent expressions and then simplify.

1. \((64)^{2/3}\)
2. \(16^{-1/2}\)
3. \((-27)^{1/3}\)

Simplify the following expressions. Your final expressions should contain no negative exponents and no parentheses.

4. \(\left(\frac{3}{5x}\right)^{-2}\)
5. \((36^{1/2}x^4)(16x^3)\)
6. \(\left(\frac{x^7y^3}{x}\right)^{1/3}\)

7. \((16a^8b^{12})^{3/4}\)
8. \(\frac{144^{1/2}x^{-3}}{(16^{3/4}x^7)^0}\)
9. \(\frac{a^{2/3}b^{-3/4}c^{7/8}}{a^{-1/3}b^{1/4}c^{1/8}}\)

Answers

Example answers are given for problems 1–3; other answers are possible.

1. \(\frac{3\sqrt{64}}{2}, (64^{2/3}), (\sqrt[3]{64})^2, 16\)
2. \(\frac{1}{(16)^{1/2}} \cdot \sqrt{16}, \frac{1}{\sqrt{16}}, \frac{1}{4}\)
3. \(((-3)^3)^{1/3}, \sqrt[3]{-27}, \sqrt[3]{-27}, -3\)
4. \(\frac{25r^2}{9}\)
5. \(96x^7\)
6. \(x^2y\)
7. \(8a^6b^9\)
8. \(\frac{12}{x^3}\)
9. \(\frac{ae^{3/4}}{b}\)