Chapter 9

AVERAGE RATE OF CHANGE

9.3.1

In previous math courses students learned that for a linear function, there is a constant rate of change which corresponds to the slope of a line. In Lesson 9.3.1, students learn how to calculate the average rate of change of a nonlinear function over a given interval. The average rate of change is calculated by determining the slope between two points on the graph of the function. If the slope represents a distance per time, then the average rate of change describes the average speed or velocity.

For additional information see the Math Notes box in Lesson 9.3.2.

Example 1

Using the data given in the table at right for an object’s position at given times, calculate the average rate of change for the following time periods:

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1.0</td>
<td>120.6</td>
</tr>
<tr>
<td>2.0</td>
<td>176.8</td>
</tr>
<tr>
<td>2.5</td>
<td>181.0</td>
</tr>
<tr>
<td>3.0</td>
<td>169.2</td>
</tr>
<tr>
<td>4.0</td>
<td>97.6</td>
</tr>
</tbody>
</table>

a. 0.0 to 1.0 seconds
b. 1.0 to 2.0 seconds
c. 2.0 to 2.5 seconds
d. 2.5 to 3.0 seconds
e. 3.0 to 4.0 seconds
f. During which time period is the object moving the fastest? When is it moving the slowest? When does it change direction?

For each time period, calculate the slope between the points corresponding to the given intervals. For example, for the time period 0.0 to 1.0 seconds the points are (0.0, 0.0) and (1.0, 120.6).

Remember to include units in the answers.

a. \( \frac{120.6 - 0}{1.0 - 0} = \frac{120.6}{1.0} = 120.6 \text{ m/s} \)
b. \( \frac{176.8 - 120.6}{2.0 - 1.0} = \frac{56.2}{1.0} = 56.2 \text{ m/s} \)
c. \( \frac{181.0 - 176.8}{2.5 - 2.0} = 8.4 \text{ m/s} \)
d. \( \frac{169.2 - 181.0}{3.0 - 2.5} = -23.6 \text{ m/s} \)
e. \( \frac{97.6 - 169.2}{4.0 - 3.0} = -71.6 \text{ m/s} \)
f. The object is moving the fastest during the time period of 0 to 1 second. It is moving the slowest (while traveling upward) between 2.0 to 2.5 seconds. It changes direction between 2.5 and 3.0 seconds.

Notice that when the object is decreasing in height (going downward) the average rate of change (velocity) is negative.
Example 2

Use the graph at right to estimate the average rate of change for the interval:

a. \( x = -1 \) to \( x = 1 \)
b. \( x = 2 \) to \( x = 3 \)
c. \( x = 4 \) to \( x = 5 \)
d. \( x = 6 \) to \( x = 7 \)
e. \( x = 8 \) to \( x = 9 \)

For each of these intervals, determine the corresponding points and then calculate the slope.

a. \((-1, -5)\) and \((1, 4)\) \[ \frac{4 - (-5)}{1 - (-1)} = 4.5 \]
b. \((2, 7)\) and \((3, 9)\) \[ \frac{9 - 7}{3 - 2} = 2 \]
c. \((4, 10)\) and \((5, 10)\) \[ \frac{10 - 10}{5 - 4} = 0 \]
d. \((6, 9)\) and \((7, 7)\) \[ \frac{7 - 9}{7 - 6} = -2 \]
e. \((8, 4)\) and \((9, 0)\) \[ \frac{0 - 4}{9 - 8} = -4 \]

Problems

Calculate the average rate of change for the given time periods using the data in the table at right.

1. 0 to 5 minutes
2. 10 to 15 minutes
3. 15 to 20 minutes
4. 25 to 30 minutes

Use the graph at right to approximate the average rate of change for the given time periods.

5. \( t = 0 \) to \( t = 1 \) s
6. \( t = 3 \) to \( t = 4 \) s
7. \( t = 4 \) to \( t = 5 \) s
8. \( t = 8 \) to \( t = 9 \) s
9. \( t = 10 \) to \( t = 11 \) s
Answers

1. 13.5 m/min  
2. 8.5 m/min  
3. −1.5 m/min  
4. −11.5 m/min  
5. ≈ 5 ft/s  
6. ≈ 2 ft/s  
7. ≈ 1.5 ft/s  
8. ≈ −2 ft/s  
9. ≈ −4.5 ft/s