Students explore inverse functions, that is, equations that “undo” the actions of functions.

For example, the function $f(x) = 3x + 1$ performs the following operations on $x$: it multiplies by 3 and then adds 1. The inverse function, named $f^{-1}(x)$, reverses the operations: it subtracts 1 and then divides by 3. Therefore, $f^{-1}(x) = \frac{x-1}{3}$.

Example 1

Determine the inverse of each function.

a. $h(x) = \frac{x-6}{3}$

b. $g(x) = 2(x + 4)$

The function in part (a) subtracts 6 from the input ($x$) and then divides the result by 3. The inverse function reverses this process. Therefore, the inverse function should multiply by 3 and then add 6. The inverse function is $h^{-1}(x) = 3x + 6$.

Test an input value in the original function: For $x = 6$, $h(6) = \frac{6-6}{3} = 0$.

Use the output value as the input to the inverse function: For $x = 0$, $h^{-1}(0) = 3(0) + 6 = 6$.

The inverse function “undoes” the original function, yielding the original input value of 6.

In part (b), the function $g(x)$ adds 4 to the input and then multiplies the result by 2. The inverse function must first divide by 2 and then subtract 4. Therefore, $g^{-1}(x) = \frac{x}{2} - 4$.

To verify this result, test an input value in the original function: $g(1) = 2(1 + 4) = 10$.

Use the output value as the input to the inverse function: $g^{-1}(10) = \frac{10}{2} - 4 = 5 - 4 = 1$.

The inverse function “undoes” the original function, yielding the original input value of 1.
Problems

Write the inverse of each of the following functions.

1. \( f(x) = 8(x - 13) \)
2. \( f(x) = -\frac{3}{4}x + 6 \)
3. \( f(x) = \frac{5(x+2)}{3} \)
4. \( f(x) = 2x + 6 \)
5. \( f(x) = \frac{3x+6}{5} \)
6. \( g(x) = \frac{x}{5} \)
7. \( g(x) = 4(x + 1) - 3 \)
8. \( j(x) = 2(x + 2) \)
9. \( h(x) = 3x - 4 \)
10. \( g(x) = 6x + 2 \)

Answers

1. \( f^{-1}(x) = \frac{x}{8} + 13 \)
2. \( f^{-1}(x) = -\frac{4}{3}x + 8 \)
3. \( f^{-1}(x) = \frac{2}{3}x - 2 \)
4. \( f^{-1}(x) = \frac{1}{2}x - 3 \)
5. \( f^{-1}(x) = \frac{5x-6}{3} \)
6. \( g^{-1}(x) = 5x \)
7. \( g^{-1}(x) = \frac{x+3}{4} - 1 \)
8. \( j^{-1}(x) = \frac{1}{2}x - 2 \)
9. \( h^{-1}(x) = \frac{1}{3}x + 4 \)
10. \( g^{-1}(x) = \frac{x}{6} - \frac{1}{3} \)