In this course, one focus is on what a solution means, both algebraically and graphically. By understanding the nature of solutions, students are able to solve equations in new and different ways. This understanding also provides opportunities to solve some challenging application problems. In Chapter 11 this knowledge is extended to solve equations and systems of equations with three variables.

Example 1

The graph of $y = (x - 5)^2 - 4$ is shown at right. Use the graph to solve each of the following equations.

a. $(x - 5)^2 - 4 = 12$

b. $(x - 5)^2 - 4 = -3$

c. $(x - 5)^2 = 4$

Solutions:

a. Add the graph of $y = 12$ which is a horizontal line to the graph of $y = (x - 5)^2 - 4$. These two graphs intersect at two points, $(1, 12)$ and $(9, 12)$. The x-coordinates of these points are the solutions to the original equation. Notice that there is no “y” in the equation in part (a). Therefore the solutions to the equation are $x = 1$ and $x = 9$.

b. Add the graph of $y = -3$ to see that the graphs intersect at $(4, -3)$ and $(6, -3)$. Therefore the solutions to the equation are $x = 4$ and $x = 6$.

c. The equation might look as if it cannot be solved with the graph, but it can. By recognizing the equation is equivalent to $(x - 5)^2 - 4 = 0$ (subtract 4 from both sides), then the graph can be used to find where the parabola crosses the line $y = 0$ (the x-axis). The graph tells us the solutions are $x = 7$ and $x = 3$. 

Example 2

Solve the equation \( \sqrt{x + 2} = 2x + 1 \) using at least two different methods. Explain your methods and the implications of the solution(s).

Solution:

One method is to use algebra to solve this equation. This involves squaring both sides and solving a quadratic equation as shown at right.

A problem arises, however, if the solutions are not checked. When each \( x \)-value is substituted back into the original equation, only one \( x \)-value checks: \( x = \frac{1}{4} \). This is the only solution.

\[
\frac{\sqrt{\frac{1}{4} + 2}}{\sqrt{\frac{9}{4}}} = \frac{2 \left( \frac{1}{4} \right) + 1}{3} \quad \sqrt{\frac{1}{4} + 2} = 2 \left( \frac{1}{4} \right) + 1 \\
\frac{\sqrt{\frac{9}{4} + 1}}{\sqrt{3}} = \frac{2 + 1}{3} \quad \sqrt{\frac{9}{4} + 1} = 2 + 1 \\
\frac{3}{2} = \frac{3}{2} \quad \sqrt{1} = -2 + 1 \\
1 = 1
\]

To see why the other solution does not work use a graph to solve the equation. The graphs of \( y = \sqrt{x + 2} \) and \( y = 2x + 1 \) are shown at right. Notice that the graphs only intersect at one point, namely \( x = \frac{1}{4} \). There is only solution to the equation; the other solution is called an extraneous solution.

Remember that a solution makes the equation true. In the original equation, this means that both sides of the equation will be equal for certain values of \( x \). Using the graphs, the solution is the \( x \)-value that has the same \( y \)-value for both functions, or the \( x \)-coordinate(s) of the point(s) at which the graphs intersect.
Example 3

Algebraically solve each system of equations below. For each system, explain what the solution (or lack thereof) tells about the graph of the system.

a. \[ y = -\frac{2}{5}x + 3 \quad y = \frac{3}{5}x - 2 \]

b. \[ y = -2(x - 2)^2 + 35 \quad y = -2x + 15 \]

c. \[ y = \frac{1}{6}x^2 - \frac{17}{3} \quad x^2 + y^2 = 25 \]

Solutions:

a. The two equations are written in “y =” form, which makes substitution the most efficient method for solving. Set the expressions on the right side of each equation equal to each other and solve for \( x \).

\[
\begin{align*}
y &= -\frac{2}{5}x + 3 \\
y &= \frac{3}{5}x - 2 \\
-\frac{2}{5}x + 3 &= \frac{3}{5}x - 2 \\
5\left(-\frac{2}{5}x + 3\right) &= 5\left(\frac{3}{5}x - 2\right) \\
-2x + 15 &= 3x - 10 \\
-5x &= -25 \\
x &= 5
\end{align*}
\]

Then substitute this value for \( x \) back into either one of the original equations to determine the value of \( y \). Finally, check that the solution satisfies both equations.

\[
\begin{align*}
y &= -\frac{2}{5}x + 3 \\
y &= \frac{3}{5}x - 2 \\
? &= \frac{3}{5}(5) - 2 \\
? &= 1 \\
1 &= -2 + 3 \checkmark \\
1 &= 3 - 2 \checkmark
\end{align*}
\]

Solution to check: \((5, 1)\)

The solution is the point \((5, 1)\), which means that the graphs of these two equations intersect at one point, the point \((5, 1)\).

b. The two equations are written in “y =” form, which means that substitution can again be used. This is shown at right.

Now substitute each \( x \)-value into either equation to calculate the corresponding \( y \)-value.

\[
\begin{align*}
x &= 6, \quad y = -2x + 15 \\
x &= -1, \quad y = -2x + 15 \\
y &= -2(6) + 15 \\
y &= -2(-1) + 15 \\
y &= -12 + 15 \\
y &= 2 + 15 \\
y &= 3 \\
y &= 17 \\
\text{Solution: } (6, 3) \\
\text{Solution: } (-1, 17)
\end{align*}
\]

Check

\[
\begin{align*}
-2(x - 2)^2 + 35 &= -2x + 15 \\
-2(x - 2)^2 &= -2x - 20 \\
-2(x^2 - 4x + 4) &= -2x - 20 \\
-2x^2 + 8x - 8 &= -2x - 20 \\
-2x^2 + 10x + 12 &= 0 \\
-2(x^2 - 5x - 6) &= 0 \\
-2(x - 6)(x + 1) &= 0 \\
-2 \neq 0, \quad x &= 6, x &= -1
\end{align*}
\]
Solution continued from previous page.

Lastly, check each point in both equations to make sure there are not any extraneous solutions.

\[
\begin{align*}
(6, 3): \quad y &= -2(x - 2)^2 + 35 \\
3 &= -2(6 - 2)^2 + 35 \\
3 &= -2(16) + 35 \checkmark \\
(6, 3): \quad y &= -2x + 15 \\
3 &= -2(6) + 15 \checkmark \\
(1, 17): \quad y &= -2(x - 2)^2 + 35 \\
17 &= -2(1 - 2)^2 + 35 \\
17 &= -2(9) + 35 \checkmark \\
(1, 17): \quad y &= -2x + 15 \\
17 &= -2(-1) + 15 \checkmark \\
\end{align*}
\]

In solving these two equations with two unknowns, two solutions were found, both of which checked in the original equations. This means that the graphs of the equations, a parabola and a line, intersect in two distinct points.

c. This system requires substitution to solve. One option is to replace \( y \) in the second equation with the right hand side of the first equation, but that would require solving an equation of degree four (an exponent of 4). Instead, rewrite the first equation without fractions in order to simplify. This is done by multiplying both sides of the equation by 6, as shown at right.

\[
y = \frac{1}{6} x^2 - \frac{17}{3}
\]

\[
6y = x^2 - 34
\]

Now, instead, replace the \( x^2 \) in the second equation with \( 6y + 34 \). Then solve.

\[
\begin{align*}
x^2 + y^2 &= 25 \\
(6y + 34) + y^2 &= 25 \\
y^2 + 6y + 34 &= 25 \\
y^2 + 6y + 9 &= 0 \\
(y + 3)(y + 3) &= 0 \\
y &= -3
\end{align*}
\]

Next, substitute this value back into either equation to find the corresponding \( x \)-value.

\[
y = -3: \quad 6y + 34 = x^2
\]

\[
6(-3) + 34 = x^2 \\
-18 + 34 = x^2 \\
16 = x^2 \\
x = \pm 4
\]

(4, -3): \quad \begin{align*}
y &= \frac{1}{6} x^2 - \frac{34}{6} \\
-3 &= \frac{1}{6} (4)^2 - \frac{34}{6} = \frac{16}{6} - \frac{34}{6} = -\frac{18}{6} \checkmark
\end{align*}

(4, -3): \quad \begin{align*}
x^2 + y^2 &= 25 \\
(4)^2 + (-3)^2 &= 16 + 9 = 25 \checkmark
\end{align*}

(-4, -3): \quad \begin{align*}
y &= \frac{1}{6} x^2 - \frac{34}{6} \\
-3 &= \frac{1}{6} (-4)^2 - \frac{34}{6} = \frac{16}{6} - \frac{34}{6} = -\frac{18}{6} \checkmark
\end{align*}

(-4, -3): \quad \begin{align*}
x^2 + y^2 &= 25 \\
(-4)^2 + (-3)^2 &= 16 + 9 = 25 \checkmark
\end{align*}

Since there are two points that make this system true, the graphs of this parabola and this circle intersect in only two points, (4, -3) and (-4, -3).
Example 4

Jo has small containers of lemonade and lime soda. She once mixed one container of lemonade with three containers of lime soda to make 17 ounces of a tasty drink. Another time, she combined five containers of lemonade with six containers of lime soda to produce 58 ounces of another splendid beverage. Given this information, how many ounces are in each small container of lemonade and lime soda?

Solution:

Solve this problem by using a system of equations. To start, let \( x \) = the number of ounces of lemonade in each small container, and let \( y \) = the number of ounces of lime soda in each of its small containers. Write an equation that describes each mixture Jo created.

The first mixture used one container (1\( x \) ounces) of lemonade and three containers (3\( y \) ounces) of lime soda for a total of 17 ounces. This can be represented as \( 1x + 3y = 17 \).

The second mixture used five containers (5\( x \) ounces) of lemonade and six containers (6\( y \) ounces) of lime soda for a total of 58 ounces. This can be represented by the equation \( 5x + 6y = 58 \).

Solve this system to determine the values of \( x \) and \( y \).

\[
\begin{align*}
x + 3y &= 17 \quad \rightarrow \quad -5x - 15y &= -85 \\
5x + 6y &= 58 \\
\end{align*}
\]

If \( y = 3 \), then:

\[
\begin{align*}
x + 3(3) &= 17 \\
x + 9 &= 17 \\
x &= 8
\end{align*}
\]

(Note: Check these values!)

Therefore each container of lemonade has 8 ounces, and each container of lime soda has 3 ounces.)
Problems

Solve each of the following systems of equations. Then explain what the solution(s) tells you about the graphs of the equations. Be sure to check your work.

1. \( x + y = 11 \)
   \( 3x - y = 5 \)
2. \( 2x - 3y = -19 \)
   \( -5x + 2y = 20 \)
3. \( 15x + 10y = 21 \)
   \( 6x + 4y = 11 \)
4. \( 8x + 2y = 18 \)
   \( -6x + y = 14 \)
5. \( 12x - 16y = 24 \)
   \( y = \frac{3}{4} x - \frac{3}{2} \)
6. \( \frac{1}{2} x - 7y = -15 \)
   \( 3x - 4y = 24 \)

The graph of \( y = \frac{1}{2} (x - 4)^2 + 3 \) is shown at right. Use the graph to solve each of the following equations. Explain how you get your answers.

7. \( \frac{1}{2} (x - 4)^2 + 3 = 3 \)
8. \( \frac{1}{2} (x - 4)^2 + 3 = 5 \)
9. \( \frac{1}{2} (x - 4)^2 + 3 = 1 \)
10. \( \frac{1}{2} (x - 4)^2 = 8 \)

Solve each equation below.

11. \( 3(x - 4)^2 + 6 = 33 \)
12. \( \frac{x}{4} + \frac{x}{5} = \frac{9x - 4}{20} \)
13. \( 3 + \left( \frac{10 - 3x}{2} \right) = 5 \)
14. \( -3\sqrt{2x - 1} + 4 = 10 \)

Solve each of the following systems of equations algebraically. What does the solution tell you about the graph of the system?

15. \( y = -\frac{2}{3} x + 7 \)
   \( 4x + 6y = 42 \)
16. \( y = (x + 1)^2 + 3 \)
   \( y = 2x + 4 \)
17. \( y = -3(x - 4)^2 - 2 \)
   \( y = -\frac{4}{7} x + 4 \)
18. \( x + y = 0 \)
   \( y = (x - 4)^2 - 6 \)

19. Adult tickets for the *Mr. Moose’s Fantasy Show on Ice* are $6.50 while a child’s ticket is only $2.50. At Tuesday night’s performance, 435 people were in attendance. The box office brought in $1667.50 for that evening. How many of each type of ticket were sold?

20. The next math test will contain 50 questions. Some will be worth three points while the rest will be worth six points. If the test is worth 195 points, how many three-point questions are there, and how many six-point questions are there?
21. Dudley’s water balloons follow the path described by the equation \( y = -\frac{8}{125} (x - 10)^2 + \frac{72}{5} \). Suppose Dudley’s nemesis, in a mad dash to save his base from total water balloon bombardment, ran to the wall and set up his launcher at its base. Dudley’s nemesis launches his balloons to follow the path \( y = -x \left( x - \frac{189}{25} \right) \) in an effort to knock Dudley’s water bombs out of the air. Is Dudley’s nemesis successful? Explain.

Answers

1. (4, 7)  
2. (-2, 5)  
3. no solution
4. \(-\frac{1}{2}, 11\)  
5. all real numbers  
6. (12, 3)
7. \( x = 4 \)
   The horizontal line \( y = 3 \) crosses the parabola at one point, at the vertex.
8. \( x = 2 \) or \( x = 6 \)
   The horizontal line \( y = 5 \) crosses the parabola at two points.
9. no real solution
   The horizontal line \( y = 1 \) does not cross the parabola.
   (Solving algebraically yields \( x = 4 \pm 2i \).)
10. \( x = 0 \) or \( x = 8 \)
    Add three to both sides to rewrite the equation as \( \frac{1}{2} (x - 4)^2 + 3 = 11 \). The horizontal line \( y = 11 \) crosses the parabola at two points.
11. \( x = 7 \) or \( x = 1 \)
12. no solution
13. \( x = 2 \)
14. no solution
15. all real numbers
    When graphed, these equations give the same line.
16. (0, 4)
    The parabola and the line intersect at one point.
17. no solution
    This parabola and this line do not intersect.
18. (2, -2) and (5, -5)
    The line and the parabola intersect twice.
19. 145 adult tickets were sold, while 290 child tickets were sold.
20. There are 35 three-point questions and 15 six-point questions on the test.
INEQUALITIES

Once the meaning of a solution is understood, it can be applied to understanding solutions of inequalities and systems of inequalities. Inequalities typically have infinitely many solutions, and students learn ways to represent such solutions. For additional information see the Math Notes boxes in Lessons 3.2.2 and 3.2.4.

Example 1

Solve each equation or inequality below. Explain what the solution for each one represents. Then explain how the equation and inequalities are related to each other.

\[ x^2 - 4x - 5 = 0 \quad x^2 - 4x - 5 < 0 \quad y \geq x^2 - 4x - 5 \]

Solution:

There are many ways to solve the equation, including graphing, factoring and using the Zero Product Property, or using the Quadratic Formula. Factoring and use the Zero Product Property to solve is shown below.

\[ x^2 - 4x - 5 = 0 \]
\[(x - 5)(x + 1) = 0 \]
\[ x = 5, \ x = -1 \]

Check:
If \( x = 5 \):
\[ (5)^2 - 4(5) - 5 = 25 - 20 - 5 = 0 \ \checkmark \]
If \( x = -1 \):
\[ (-1)^2 - 4(-1) - 5 = 1 + 4 - 5 = 0 \ \checkmark \]

The second quadratic is an inequality. To solve this inequality, utilize a number line to emphasize what the solution represents.

Start by solving the related quadratic equation (which was done above). \( x^2 - 4x - 5 = 0 \)

The solutions to the equation are \( x = 5 \) and \( x = -1 \).

By placing these two points on a number line, they act as boundary points, dividing the number line into three regions. Since the original inequality is strictly “less than,” use open circles for the boundary points.

Choose any number in each of the regions to see if the number will make the original inequality true or false. Solutions will make the inequality true. Note: You only need to check one point in each region.

Solution continues on next page →
Choosing a point in each region and substituting into the inequality gives:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Result</th>
<th>Condition</th>
<th>Result</th>
<th>Condition</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = -2 )</td>
<td>((-2)^2 - 4(-2) - 5 &lt; 0)</td>
<td>( x = 0 )</td>
<td>((0)^2 - 4(0) - 5 &lt; 0)</td>
<td>( x = 7 )</td>
<td>((7)^2 - 4(7) - 5 &lt; 0)</td>
</tr>
<tr>
<td>( (-2)^2 - 4(-2) - 5 &lt; 0 )</td>
<td>( 4 + 8 - 5 &lt; 0 )</td>
<td>( 0 - 0 - 5 &lt; 0 )</td>
<td>( 49 - 28 - 5 &lt; 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 7 &lt; 0 )</td>
<td>false</td>
<td>( -5 &lt; 0 )</td>
<td>true</td>
<td>( 16 &lt; 0 )</td>
<td>false</td>
</tr>
</tbody>
</table>

Highlight the region on the number line that makes the inequality true, as shown at right. This solution can also be represented algebraically as \(-1 < x < 5\).

The last inequality in the example has a \( y \). Having both \( x \) and \( y \) means an \( xy \)-coordinate graph needs to be used to show the solutions. Graph the parabola using a solid line because the original inequality is “greater than or equal to”. The graph of the parabola at right divides the plane into two regions: the part within the “bowl” of the parabola—the interior—and the region outside the parabola. The points on the parabola represent where \( y = x^2 - 4x - 5 \).

Test a point from one of the regions to check whether it will make the inequality true or false. As before, we are looking for the “true” region.

The point \((0, 0)\) is an easy point to use.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Result</th>
<th>Condition</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 \geq (0)^2 - 4(0) - 5 )</td>
<td>True! Therefore the region containing the point ((0, 0)) is the solution. This means any point chosen in this region, the “bowl” of the parabola, will make the inequality true.</td>
<td>( 0 \geq 0 - 0 - 5 )</td>
<td>( 0 \geq -5 )</td>
</tr>
</tbody>
</table>

To illustrate that this region is the solution, we shade this region of the graph.

To see how these equations and inequalities are related, examine the graph of the parabola. Where are the zeros? Where are the \( y \)-values of the parabola negative? The zeros are the \( x \)-intercepts on the graph, or \( x = -1 \) and \( x = 5 \), which you determined by solving the equation. The graph is negative when it dips below the \( x \)-axis, and this happens when \( x \) is between \(-1 \) and \( 5 \). Solving the first inequality answered this question as well. Therefore, the graph could have answered the first two parts quickly.
Example 2

Han and Lea have been building jet roamers and pod racers. Each jet roamer requires one jet pack and three crystallic fuel tanks, while each pod racer requires two jet packs and four crystallic fuel tanks. Han and Lea’s suppliers can only produce 100 jet packs and 270 fuel tanks each week, and due to manufacturing conditions, Han and Lea can build no more than 30 pod racers each week. Each jet roamer makes a profit of 1 tig (their form of currency) while each pod racer makes a profit of 4 tigs.

a. If Han and Lea receive an order for 12 jet roamers and 22 pod racers, how many of each part will they need to fill this order? If they can fill this order, how many tigs will they make?

b. Write a list of constraints, an inequality for each constraint, and sketch a graph showing all inequalities with the points of intersection labeled. How many jet roamers and pod racers should Han and Lea build to maximize their profits?

Solution:

This problem is an example of a **linear programming** problem, and although the name might conjure up images of computer programming, these problems are not done on a computer. This problem can be solved by creating a system of inequalities that, when graphed, creates a **feasibility region**. This region contains the solution for the number of jet roamers and pod racers Han and Lea should make to maximize their profit.

a. Jet packs: \(1(12) + 2(22) = 12 + 44 = 56\)
   Fuel tanks: \(3(12) + 4(22) = 36 + 88 = 124\)
   Each result is within the constraints, so it is possible for Han and Lea to fill this order. If they do, they will make \(1(12) + 4(22) = 12 + 88 = 100\) tigs.

b. Begin by defining the variables. Let \(x\) = the number of jet roamers Han and Lea will make, and \(y\) = the number of pod racers.

   \(x \geq 0,\) and \(y \geq 0\) because a negative number of items cannot be produced.
   A jet roamer requires one jet pack while a pod racer requires two. There are only 100 jet packs available each week, so \(x + 2y \leq 100.\)
   Each jet roamer requires three crystallic fuel tanks and each pod racer requires four. This translates into the inequality \(3x + 4y \leq 270\) since only 270 fuel tanks are available each week. Lastly, since Han and Lea cannot make more than 30 pod racers, we can write \(y \leq 30.\)

These inequalities are all shown on the graph at right. The region common to all constraints is shaded. This is the **feasibility region** because choosing a point in this shaded area gives you a combination of jet roamers and pod racers that Han and Lea can produce under the given restraints.

*Solution continues on next page →*
Solution continued from previous page.

The equation used to calculate the profit is \( P = 1x + 4y \).

To maximize profits, test all the vertices of the feasibility region in the profit equation. These points are \((0, 0), (0, 30), (40, 30), (70, 15), \) and \((90, 0)\).

\[
\begin{align*}
(0, 0): & \quad P = 1(0) + 4(0) = 0 \\
(0, 30): & \quad P = 1(0) + 4(30) = 120 \\
(40, 30): & \quad P = 1(40) + 4(30) = 160 \\
(70, 15): & \quad P = 1(70) + 4(15) = 130 \\
(90, 0): & \quad P = 1(90) + 4(0) = 90
\end{align*}
\]

The greatest profit is 160 tigs when Han and Lea build 40 jet roamers and 30 pod racers.

Problems

Graph the following system of inequalities.

1. \[
\begin{align*}
y &< \frac{1}{2}x + 6 \\
y &> -\frac{1}{2}x + 6 \\
x &< 12
\end{align*}
\]

2. \[
\begin{align*}
x + y &< 10 \\
x + y &> 4 \\
x &< 2x \\
y &> y
\end{align*}
\]

3. \[
\begin{align*}
y &\leq 3x + 4 \\
y &\geq -\frac{1}{4}x + 8 \\
y &\geq -\frac{1}{3}x + 4 \\
y &\geq 5x - 6
\end{align*}
\]

4. \[
\begin{align*}
3x + 4y &< 12 \\
y &> (x + 1)^2 - 4
\end{align*}
\]

5. \[
\begin{align*}
y &< -\frac{3}{4}(x - 1)^2 + 6 \\
y &> x - 7
\end{align*}
\]

6. \[
\begin{align*}
y &< (x + 2)^3 \\
y &> x^2 + 3x
\end{align*}
\]

For each of the following problems, write a system of inequalities that when graphed will produce the shaded region.

7. 

8. 

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9. Ramon and Thea are considering opening their own business. They wish to make and sell alien dolls they call Hauteans and Zotions. Each Hautean sells for $1.00 while each Zotion sells for $1.50. They can make up to 20 Hauteans and 40 Zotions, but no more than 50 dolls total. When Ramon and Thea go to city hall to get a business license, they find there are a few more restrictions on their production. The number of Zotions (the more expensive item) can be no more than three times the number of Hauteans (the cheaper item). How many of each doll should Ramon and Thea make to maximize their profit? What will the profit be?

10. Sam and Emma are plant managers for the Sticky Chewy Candy Company that specializes in delectable gourmet candies. Their two most popular candies are Chocolate Chews and Peanut and Jelly Jimmies. Each batch of Chocolate Chews takes 1 teaspoon of vanilla while each batch of the Peanut and Jelly Jimmies uses two teaspoons of vanilla. They have at most 20 teaspoons of vanilla on hand as they use only the freshest of ingredients. The Chocolate Chews use two teaspoons of baking soda while the Peanut and Jelly Jimmies use three teaspoons of baking soda. They only have 36 teaspoons of baking soda on hand. Because of production restrictions, they can make no more than 15 batches of Chocolate Chews and no more than 7 batches of Peanut and Jelly Jimmies. Sam and Emma have been given the task of determining how many batches of each candy they should produce if they make $3.00 profit for each batch of Chocolate Chews and $2.00 for each batch of Peanut and Jelly Jimmies. Help them out by writing the inequalities described here, graphing the feasibility region, and determining their maximum profit.

Answers

1. ![Graph 1]
2. ![Graph 2]
3. ![Graph 3]
4. ![Graph 4]
5. ![Graph 5]
6. ![Graph 6]
7. \[ y \leq \frac{1}{3} x + 4 \]
   \[ y \leq -x + 8 \]
   \[ y \geq -\frac{1}{2} x + 4 \]

8. \[ y \geq (x - 6)^2 - 5 \]
   \[ y \leq 0 \]

9. The graph of the feasibility region is shown at right. The inequalities are \( x \geq 0, y \geq 0, x + y \leq 50, x \leq 20, y \leq 40, \) and \( y \leq 3x, \) where \( x = \) number of Hauteans and \( y = \) number of Zotions. The profit is given by \( P = x + 1.5y. \) Maximum profit seems to occur at point \( A \) (12.5, 37.5), but there is a problem with this point. Ramon and Thea cannot make a half of a doll (or at least that does not seem possible). Try these nearby points: (12, 37), (12, 38), (13, 37), and (13, 38). The point that gives maximum profit and is still in the feasibility region is (13, 37). They should make 13 Hautean and 37 Zotion dolls for a profit of $68.50.

10. The graph of the feasibility region is shown at right. The inequalities are \( x \geq 0, y \geq 0, y \leq 7, x \leq 15, x + 2y \leq 20, \) and \( 2x + 3y \leq 36, \) where \( x = \) number of Chocolate Chews and \( y = \) number of Peanut and Jelly Jimmies. The profit is given by \( P = 3x + 2y. \) The point that seems to give the maximum profit is (15, 2.5) but this only works if half batches can be made. Instead, choose the point (15, 2) which means Sam and Emma should make 15 batches of Chocolate Chews and 2 batches of Peanut and Jelly Jimmies. Their profit will be $49.00.
1. If \( \frac{x+4}{12} = \frac{4}{3} \), then \( x \) equals:
   a. 3  b. 6  c. 8  d. 10  e. 12

2. What is the least of three consecutive integers whose sum is 21?
   a. 5  b. 6  c. 7  d. 8  e. 9

3. Juanita has stocks, bonds, and t-bills for investments. The number of t-bills she has is one more than the number of stocks, and the number of bonds is three times the number of t-bills. Which of the following could be the total number of investments?
   a. 16  b. 17  c. 18  d. 19  e. 20

4. Through how many degrees would the minute hand of a clock turn from 5:20 p.m. to 5:35 p.m. the same day?
   a. 15°  b. 30°  c. 45°  d. 60°  e. 90°

5. The length of a rectangle is six times its width. If the perimeter of the rectangle is 56, what is the width of the rectangle?
   a. 4  b. 7  c. 8.5  d. 18  e. 24

6. If \( m > 1 \) and \( m^nm^5 = m^{15} \), then what does \( n \) equal?

7. In the triangle at right, what is the value of \( a + b + c + d \)?

8. If \( x \) and \( y \) are positive integers, \( x + y < 12 \), and \( x > 4 \), what is the greatest possible value for \( x - y \)?

9. If \( (2x^2 + 5x + 3)(2x + 4) = ax^3 + bx^2 + cx + d \) for all values of \( x \) what does \( c \) equal?

10. Four lines intersect in one point creating eight congruent adjacent angles. What is the measure of one of these angles?
Answers

1. E
2. B
3. D
4. E
5. A
6. 10
7. $280^\circ$
8. 9
9. 26
10. $45^\circ$