MULTIPLICATION AND DIVISION
OF RATIONAL EXPRESSIONS

Multiplication or division of rational expressions follows the same procedures used with numerical fractions. However, it is often necessary to factor the polynomials in order to simplify them. As in the previous section, remember that simplification assumes that the denominator is not equal to zero. For additional information, see examples 3 and 4 in the Math Notes box in Lesson 11.1.3.

Example 1
Multiply \( \frac{x^2 + 6x}{(x + 6)^2} \cdot \frac{x^2 + 7x + 6}{x^2 - 1} \) and simplify the result.

After factoring, the expression becomes:

\[
\frac{x(x + 6)}{(x + 6)(x + 6)} \cdot \frac{(x + 6)(x + 1)}{(x + 1)(x - 1)}
\]

After multiplying, reorder the factors:

\[
\frac{(x + 6)}{(x + 6)} \cdot \frac{(x + 6)}{(x + 6)} \cdot \frac{x}{(x - 1)} \cdot \frac{(x + 1)}{(x + 1)}
\]

Since \( \frac{(x + 6)}{(x + 6)} = 1 \) and \( \frac{(x + 1)}{(x + 1)} = 1 \), simplify:

\[
1 \cdot \frac{x}{x - 1} \Rightarrow \frac{x}{x - 1}
\]

Example 2
Divide \( \frac{x^2 - 4x - 5}{x^2 - 4x + 4} + \frac{x^2 - 2x - 15}{x^2 + 4x - 12} \) and simplify the result.

First change to a multiplication expression by inverting (flipping) the second fraction:

\[
\frac{x^2 - 4x - 5}{x^2 - 4x + 4} \cdot \frac{x^2 - 2x - 15}{x^2 + 4x - 12}
\]

After factoring, the expression is:

\[
\frac{(x - 5)(x + 1)}{(x - 2)(x - 2)} \cdot \frac{(x + 6)(x - 2)}{(x - 5)(x + 3)}
\]

Reorder the factors (if you need to):

\[
\frac{(x - 5)}{(x - 5)} \cdot \frac{(x - 2)}{(x - 2)} \cdot \frac{(x + 1)}{(x - 2)} \cdot \frac{(x + 6)}{(x + 3)}
\]

Since \( \frac{(x - 5)}{(x - 5)} = 1 \) and \( \frac{(x - 2)}{(x - 2)} = 1 \), simplify:

\[
\frac{(x + 1)}{(x - 2)} \cdot \frac{(x + 6)}{(x + 3)} \text{ or } \frac{(x + 1)(x + 6)}{(x - 2)(x + 3)}
\]
Problems

Multiply or divide each pair of rational expressions. Simplify the result. Assume the denominator does not equal to zero.

1. \(\frac{x^2 + 5x + 6}{x^2 - 4x} \cdot \frac{4x}{x + 2}\)
2. \(\frac{x^2 - 2x}{x^2 - 4x + 4} \div \frac{4x^2}{x - 2}\)
3. \(\frac{x^2 - 16}{(x-4)^2} \div \frac{x^2 - 3x - 18}{x^2 - 2x - 24}\)
4. \(\frac{x^2 - x - 6}{x^2 + 3x - 10} \cdot \frac{x^2 + 2x - 15}{x^2 - 6x + 9}\)
5. \(\frac{x^2 - x - 6}{x^2 - x - 20} \div \frac{x^2 + 6x + 8}{x^2 - x - 6}\)
6. \(\frac{x^2 - x - 30}{x^2 + 13x + 40} \div \frac{x^2 + 11x + 24}{x^2 - 9x + 18}\)
7. \(\frac{15 - 5x}{x^2 - x - 6} \div \frac{5x}{x^2 + 6x + 8}\)
8. \(\frac{17x + 119}{x^2 + 5x - 14} \div \frac{9x - 1}{x^2 - 3x + 2}\)
9. \(\frac{2x^2 - 5x - 3}{3x^2 - 10x + 3} \div \frac{9x^2 - 1}{4x^2 + 4x + 1}\)
10. \(\frac{x^2 - 1}{x^2 - 6x - 7} \div \frac{x^3 + x^2 - 2x}{x - 7}\)
11. \(\frac{3x - 21}{x^2 - 49} \div \frac{3x}{x^2 + 7x}\)
12. \(\frac{x^2 - y^2}{x + y} \div \frac{1}{x - y}\)
13. \(\frac{y^2 - y}{w^2 - y^2} \div \frac{y^2 - 2y + 1}{1 - y}\)
14. \(\frac{y^2 - y - 12}{y + 2} \div \frac{y - 4}{y^2 - 4y - 12}\)
15. \(\frac{x^2 + 7x + 10}{x + 2} \div \frac{x^2 + 2x - 15}{x + 2}\)

Answers

1. \(\frac{4(x + 3)}{x - 4}\)
2. \(\frac{1}{4x}\)
3. \(\frac{x + 3}{x - 4}\)
4. \(\frac{x + 2}{x - 2}\)
5. \(\frac{x + 2}{x - 5}\)
6. \(\frac{x + 3}{x - 3}\)
7. \(\frac{-x - 4}{x}\)
8. \(\frac{17(x - 1)}{9x - 1}\)
9. \(\frac{3x + 1}{2x + 1}\)
10. \(\frac{1}{x(x + 2)}\)
11. 1
12. 1
13. \(\frac{-y}{w^2 - y^2}\)
14. \((y + 3)(y - 6)\)
15. \(\frac{x + 2}{x - 3}\)