LOGARITHMS

In this section students explore the inverse of an exponential function. Although the graph of the inverse of an exponential function can be created by reflecting the graph across the line \( y = x \), students cannot yet write the equation of this inverse function. Writing the equation requires the introduction of a new function, the logarithm.

For additional information see the Math Notes box in Lesson 5.2.4.

Example 1

Determine each of the missing values below and then justify your answer by writing the equation in its equivalent exponential form.

a. \( \log_5(25) = ? \)  
b. \( \log_7(?) = 3 \)  
c. \( \log_2 \left( \frac{1}{8} \right) = ? \)

In part (a), \( \log_5(25) \), is asking “What exponent is needed to raise the base 5 to, to get 25?” This question can be translated into an equation \( 5^x = 25 \). By phrasing it this way, the answer is more apparent: 2. This is true because \( 5^2 = 25 \).

Part (b) can be rephrased as \( 7^3 = ? \). The answer is 343.

Part (c) asks “2 to what exponent gives \( \frac{1}{8} \)” or \( 2^y = \frac{1}{8} \). The answer is −3 because \( 2^{-3} = \frac{1}{2^3} = \frac{1}{8} \).
Example 2

The graph of \( y = \log(x) \) is shown at right. Use this “parent graph” to graph each of the following equations. Describe how each graph is transformed from the parent graph. Note: When a logarithm is written without a base, as in \( y = \log(x) \) and the log key used on a calculator, the base is 10.

\[
\begin{align*}
\text{y} & = \log(x - 4) & \text{y} & = 6\log(x) + 3 & \text{y} & = -\log(x)
\end{align*}
\]

The logarithm function follows the same rules for transforming its graphs as other functions. The parent graph \( y = \log(x) \) is shown in gray at right.

\( y = \log(x - 4) \) shifts the parent graph 4 units to the right.

\( y = 6\log(x) + 3 \) shifts the parent graph up 3 units, but it is also vertically stretched by a factor of 6.

\( y = -\log(x) \) is reflected vertically or across the \( x \)-axis.

Problems

Rewrite each logarithmic equation as an exponential equation and vice versa.

1. \( 2 = \log_5(x) \)
2. \( 3 = \log_2(x) \)
3. \( x = \log_5(30) \)
4. \( 4^x = 80 \)
5. \( \left( \frac{1}{2} \right)^x = 64 \)
6. \( x^3 = 343 \)
7. \( 5^x = \frac{1}{125} \)
8. \( \log(32) = x \)
9. \( 11^3 = x \)
10. \( -4 = \log_x \left( \frac{1}{16} \right) \)

What is the value of \( x \) in each equation below? If necessary, rewrite the expression in the equivalent exponential equation to verify your answer.

11. \( 4 = \log_5(x) \)
12. \( 2 = \log_9(x) \)
13. \( 6 = \log(x) \)
14. \( 81 = 9^x \)
15. \( \left( \frac{1}{3} \right)^x = 243 \)
16. \( 6^x = 7776 \)
17. \( 7^x = \frac{1}{49} \)
18. \( \log_2(32) = x \)
19. \( \log_{11}(x) = 3 \)
20. \( \log_5 \left( \frac{1}{125} \right) = x \)

Graph each of the following equations.

21. \( y = \log(x + 2) \)
22. \( y = 3\log(x - 7) + 5 \)
23. \( y = -\log(x - 4) \)
24. \( y = \log(x) - 5 \)
Answers
1. $4^2 = x$  
2. $2^3 = x$  
3. $5^x = 30$  
4. $\log_4(80) = x$
5. $\log_{1/2}(64) = x$  
6. $\log_x(343) = 3$  
7. $\log_5\left(\frac{1}{125}\right) = x$  
8. $10^x = 32$
9. $\log_{11}(x) = 3$  
10. $x^{-4} = \frac{1}{16}$  
11. $x = 625$  
12. $x = 81$
13. $x = 1,000,000$  
14. $x = 2$  
15. $x = -5$  
16. $x = 5$
17. $x = -2$  
18. $x = 5$  
19. $x = 1,331$  
20. $x = -3$

21. 
22. 
23. 
24.