Chapter 1: Problem Solving

CHAPTER 1: PROBLEM SOLVING

<table>
<thead>
<tr>
<th>Date:</th>
<th>Learning Log Title:</th>
<th>Lesson:</th>
</tr>
</thead>
</table>

© 2011, 2013 CPM Educational Program. All rights reserved.
<table>
<thead>
<tr>
<th>Date:</th>
<th>Learning Log Title:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson:</td>
<td></td>
</tr>
<tr>
<td>Date:</td>
<td>Learning Log Title:</td>
</tr>
<tr>
<td>------</td>
<td>---------------------</td>
</tr>
<tr>
<td>Lesson:</td>
<td></td>
</tr>
</tbody>
</table>
MATH NOTES

FRACTION $\leftrightarrow$ DECIMAL $\leftrightarrow$ PERCENT

The Representations of a Portion web diagram below illustrates that fractions, decimals, and percents are different ways to represent a portion of a number. Portions can also be represented in words, such as “four fifths” or “twelve fifteenths,” or with diagrams.

![Representations of a Portion](image)

The examples below show how to convert from one form to another.

**Decimal to percent:**
Multiply the decimal by 100.

\[(0.34)(100) = 34\%\]

**Fraction to percent:**
Set up an equivalent fraction using 100 as the denominator. The numerator is the percent.

\[
\frac{\frac{4}{5} \cdot \frac{30}{20}}{100} = \frac{80}{100} = 80\%
\]

**Terminating decimal to fraction:**
Make the digits the numerator. Use the decimal place value as the denominator. Simplify as needed.

\[0.2 = \frac{2}{10} = \frac{1}{5}\]

**Percent to decimal:**
Divide the percent by 100.

\[78.6\% = 78.6 \div 100 = 0.786\]

**Percent to fraction:**
Use 100 as the denominator. Use the number in the percent as the numerator. Simplify as needed.

\[22\% = \frac{22}{100} = \frac{11}{50}\]

**Fraction to decimal:**
Divide the numerator by the denominator. If sets of digits repeat, then write just one of the repeating sets and place a “bar” over it.

\[
\frac{\frac{3}{8}}{3} = 0.375
\]

\[
\frac{\frac{70}{99}}{70 + 99} = 0.707070 \ldots = 0.70
\]
AXES, QUADRANTS, AND GRAPHING ON AN XY-COORDINATE GRAPH

Coordinate axes on a flat surface are formed by drawing vertical and horizontal number lines that meet at 0 on each number line and form a right angle (90°). The x-axis and y-axes help define points on a graph (called a “Cartesian Plane”). The x-axis is horizontal, while the y-axis is vertical. The x- and y-axes divide the graphing area into four sections called quadrants.

Numerical data can be graphed on a plane using points. Points on the graph are identified by two numbers in an ordered pair written as (x, y). The first number is the x-coordinate of the point, and the second number is the y-coordinate. The point (0, 0) is called the origin.

To locate the point (3, 2) on an xy-coordinate graph, go three units from the origin to the right to 3 on the horizontal axis and then, from that point, go 2 units up (using the y-axis scale). To locate the point (−2, −4), go 2 units from the origin to the left to −2 on the horizontal axis and then 4 units down (using the y-axis scale).
**WRITING EQUATIONS USING THE 5-D PROCESS**

The **5-D Process** is an organized method to help write equations and solve problems. The D’s stand for Describe/Draw, Define, Do, Decide, and Declare. An example of this work is shown below.

**Example Problem:** The base of a rectangle is 13 centimeters longer than the height. If the perimeter is 58 centimeters, find the base and the height of the rectangle.

**Describe/Draw:** The shape is a rectangle, and we are looking at the perimeter.

**Define**

<table>
<thead>
<tr>
<th>Height (trial)</th>
<th>Base (height + 13)</th>
<th>Perimeter 2(base) + 2(height)</th>
<th>Decide</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10 + 13 = 23</td>
<td>2(23) + 2(10) = 66</td>
<td>58</td>
</tr>
<tr>
<td><strong>Trial 1:</strong></td>
<td></td>
<td></td>
<td>66 is too high</td>
</tr>
</tbody>
</table>

Use any trial value.

Use the relationships stated in the problem to determine the values of the other quantities (such as base and perimeter).

Let $x$ represent the height in cm

$x$ | $x + 13$ | $2(x) + 2(x + 13)$ | $2(x) + 2(x + 13) = 58$

Now use your algebra skills to solve the equation.

**Declare:** The base is 21 centimeters, and the height is 8 centimeters.

If you do not write an equation, you can solve the problem by making more trials until you find the answer.
**PROPORTIONAL RELATIONSHIPS**

A proportional relationship can be seen in a table: if one quantity is multiplied by an amount, the corresponding quantity is multiplied by the same amount. On a graph, a proportional relationship is linear and goes through the origin.

Proportional example: Three pounds of chicken costs $7.00. Below, other values are shown in the table and plotted on the graph.

<table>
<thead>
<tr>
<th>Pounds</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td>0</td>
<td>7</td>
<td>14</td>
<td>21</td>
<td>28</td>
</tr>
</tbody>
</table>

The relationship between pounds and cost is proportional.

Non-proportional example: The video arcade costs $5.00 to enter and $1.00 per game.

<table>
<thead>
<tr>
<th>Games</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

The relationship between games and cost is *not* proportional. For example, someone who plays four games ($9) does not pay twice as much as someone who played two games ($7). There is no multiplier for the relationship. The graph does not go through the origin.
SOLVING PROPORTIONS

If a relationship is known to be proportional, ratios from the situation are equal. An equation stating that two ratios are equal is called a proportion. Some examples of proportions are:

\[
\frac{6 \text{ mi}}{2 \text{ hr}} = \frac{9 \text{ mi}}{3 \text{ hr}} \quad \frac{5}{7} = \frac{50}{70}
\]

Setting up a proportion is one strategy for solving for an unknown part of one ratio. For example, if the ratios \(\frac{9}{2}\) and \(\frac{x}{16}\) are equal, setting up the proportion \(\frac{x}{16} = \frac{9}{2}\) allows you to solve for \(x\).

Giant One: One way to solve this proportion is by using a Giant One to find the equivalent ratio. In this case, since 16 is 2 times 8, you create the Giant One shown at right.

\[
\frac{x}{16} = \frac{9}{2} \quad \frac{x}{16} = \frac{9 \cdot 8}{2 \cdot 8}
\]

which shows that \(x = 72\)

Undoing Division: Another way to solve the proportion is to think of the ratio \(\frac{x}{16}\) as, “\(x\) divided by 16.” To solve for \(x\), use the inverse operation of division, which is multiplication. Multiplying both sides of the proportional equation by 16 “undoes” the division.

\[
\left(\frac{16}{1}\right)\frac{x}{16} = \frac{9}{2} \left(\frac{16}{1}\right)
\]

which shows that \(x = 72\)

Cross-Multiplication: This method of solving the proportion is a shortcut for using a Fraction Buster (multiplying each side of the equation by the denominators).

\[
\begin{align*}
\text{Fraction Buster} & \quad \text{Cross-Multiplication} \\
\frac{x}{16} = \frac{9}{2} & \quad \frac{x}{16} = \frac{9}{2} \\
2 \cdot 16 \cdot \frac{x}{16} = \frac{9}{2} \cdot 2 \cdot 16 & \quad \frac{x}{16} \cdot 16 = \frac{9}{2} \\
2 \cdot x = 9 \cdot 16 & \quad 2 \cdot x = 9 \cdot 16 \\
x = 144 & \quad 2x = 144 \\
x = 72 & \quad x = 72
\end{align*}
\]