CHAPTER 2: SIMPLIFYING WITH VARIABLES

Date:  
Lesson:  
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MATH NOTES

NON-COMMENSURATE

Two measurements are called non-commensurate if no combination of one measurement can equal a combination of the other. For example, your algebra tiles are called non-commensurate because no combination of unit squares will ever be exactly equal to a combination of x-tiles (although at times they may appear close in comparison). In the same way, in the example below, no combination of x-tiles will ever be exactly equal to a combination of y-tiles.

MATHEMATICS VOCABULARY

Variable: A letter or symbol that represents one or more numbers.

Expression: A combination of numbers, variables, and operation symbols. For example, $2x + 3(5 - 2x) + 8$. Also, $5 - 2x$ is a smaller expression within the larger expression.

Term: Parts of the expression separated by addition and subtraction. For example, in the expression $2x + 3(5 - 2x) + 8$, the three terms are $2x$, $3(5 - 2x)$, and $8$. The expression $5 - 2x$ has two terms, 5 and $-2x$.

Coefficient: The numerical part of a term. In the expression $2x + 3(5 - 2x) + 8$, for example, 2 is the coefficient of $2x$. In the expression $7x - 15x^2$, both 7 and 15 are coefficients.

Constant term: A number that is not multiplied by a variable. In the expression $2x + 3(5 - 2x) + 8$, the number 8 is a constant term. The number 3 is not a constant term, because it is multiplied by a variable inside the parentheses.

Factor: Part of a multiplication expression. In the expression $3(5 - 2x)$, 3 and $5 - 2x$ are factors.
Notes:

**COMBINING LIKE TERMS**

Combining tiles that have the same area to write a simpler expression is called **combining like terms**. See the example shown at right.

When you are not working with actual tiles, it can help to picture the tiles in your mind. You can use these images to combine the terms that are the same. Here are two examples:

Example 1:
\[2x^2 + xy + y^2 + x + 3 + x^2 + 3xy + 2 \Rightarrow 3x^2 + 4xy + y^2 + x + 5\]

Example 2:
\[3x^2 - 2x + 7 - 5x^2 + 3x - 2 \Rightarrow -2x^2 + x + 5\]

A **term** is an algebraic expression that is a single number, a single variable, or the product of numbers and variables. The simplified algebraic expression in Example 2 above contains three terms. The first term is \(-2x^2\), the second term is \(x\), and the third term is 5.
EVALUATING EXPRESSIONS AND THE ORDER OF OPERATIONS

To evaluate an algebraic expression for particular values of the variables, replace the variables in the expression with their known numerical values and simplify. Replacing variables with their known values is called substitution. An example is provided below.

Evaluate $4x - 3y + 7$ for $x = 2$ and $y = 1$.

Replace $x$ and $y$ with their known values of 2 and 1, respectively, and simplify.

$4(2) - 3(1) + 7$

$= 8 - 3$

$= 12$

When evaluating a complex expression, you must remember to use the Order of Operations that mathematicians have agreed upon. As illustrated in the example below, the Order of Operations is:

Original expression:

$4x - 3y + 7$

Circle expressions that are grouped within parentheses or by a fraction bar:

Simplify within circled terms using the Order of Operations:

- Evaluate exponents.
- Multiply and divide from left to right.
- Combine terms by adding and subtracting from left to right.

Circle the remaining terms:

Simplify within circled terms using the Order of Operations as described above.
**COMMUTATIVE PROPERTIES**

The **Commutative Property of Addition** states that when *adding* two or more number or terms together, order is not important. That is:

\[ a + b = b + a. \]

For example, \( 2 + 7 = 7 + 2 \)

The **Commutative Property of Multiplication** states that when *multiplying* two or more numbers or terms together, order is not important. That is:

\[ a \cdot b = b \cdot a. \]

For example, \( 3 \cdot 5 = 5 \cdot 3 \)

However, *subtraction* and *division* are **not** commutative, as shown below.

\[
\begin{align*}
7 - 2 & \neq 2 - 7 \text{ since } 5 \neq -5 \\
50 \div 10 & \neq 10 \div 50 \text{ since } 5 \neq 0.2
\end{align*}
\]

**Simplifying an Expression (“Legal Moves””)**

Three common ways to simplify or alter expressions on an Expression Mat are illustrated below.

- **Removing an equal number of opposite tiles** that are in the same region. For example, the positive and negative tiles in the same region at right combine to make zero.

- **Flipping a tile to move it out of one region** into the opposite region (i.e., finding its opposite). For example, the tiles in the “–” region at right can be flipped into the “+” region.

- **Removing an equal number of identical tiles** from both the “–” and the “+” regions. This strategy can be seen as a combination of the two methods above, since you could first flip the tiles from one region to another and then remove the opposite pairs.
**ASSOCIATIVE AND IDENTITY PROPERTIES**

The **Associative Property of Addition** states that when *adding* three or more number or terms together, grouping is not important. That is:

\[(a + b) + c = a + (b + c)\]  
For example, \((5 + 2) + 6 = 5 + (2 + 6)\)

The **Associative Property of Multiplication** states that when *multiplying* three or more numbers or terms together, grouping is not important. That is:

\[(a \cdot b) \cdot c = a \cdot (b \cdot c)\]  
For example, \((5 \cdot 2) \cdot 6 = 5 \cdot (2 \cdot 6)\)

However, **subtraction** and **division** are not associative, as shown below.

\[(5 - 2) - 3 \neq 5 - (2 - 3)\]  
since \(0 \neq 6\)

\[(20 + 4) + 2 \neq 20 + (4 + 2)\]  
since \(2.5 \neq 10\)

The **Identity Property of Addition** states that adding zero to any expression gives the same expression. That is:

\[a + 0 = a\]  
For example, \(6 + 0 = 6\)

The **Identity Property of Multiplication** states that multiplying any expression by one gives the same expression. That is:

\[1 \cdot a = a\]  
For example, \(1 \cdot 6 = 6\)

**INVERSE PROPERTIES**

The **Additive Inverse Property** states that for every number \(a\) there is a number \(-a\) such that \(a + (-a) = 0\). A common name used for the additive inverse is the **opposite**. That is, \(-a\) is the opposite of \(a\). For example, \(3 + (-3) = 0\) and \(-5 + 5 = 0\).

The **Multiplicative Inverse Property** states that for every nonzero number \(a\) there is a number \(\frac{1}{a}\) such that \(a \cdot \frac{1}{a} = 1\). A common name used for the multiplicative inverse is the **reciprocal**. That is, \(\frac{1}{a}\) is the reciprocal of \(a\). For example, \(6 \cdot \frac{1}{6} = 1\).
USING AN EQUATION MAT

An **Equation Mat** can help you visually represent an equation with algebra tiles.

For example, the equation $2x - 1 - (-x + 3) = 6 - 2x$ can be represented by the Equation Mat below. (Note that there are other possible ways to represent this equation correctly on the Equation Mat.)