## Chapter 3: Graphs and Equations

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**Math Notes**

**Patterns in Nature**

Patterns are everywhere, especially in nature. One famous pattern that appears often is called the Fibonacci Sequence, a sequence of numbers that starts 1, 1, 2, 3, 5, 8, 13, 21, …

The Fibonacci numbers appear in many different situations in nature. For example, the number of petals on a flower is often a Fibonacci number, and the number of seeds on a spiral from the center of a sunflower is, too.

To learn more about Fibonacci numbers, search the Internet or check out a book from your local library. The next time you look at a flower, look for Fibonacci numbers!

**Discrete Graphs**

When a graph of data is limited to a set of separate, non-connected points, that relationship is called discrete. For example, consider the relationship between the number of bicycles parked at your school and the number of bicycle wheels. If there is one bicycle, it has two wheels. Two bicycles have four wheels, while three bicycles have six wheels. However, there cannot be 1.3 or 2.9 bicycles. Therefore, this data is limited because the number of bicycles must be a whole number, such as 0, 1, 2, 3, and so on.

When graphed, a discrete relationship looks like a collection of unconnected points. See the example of a discrete graph above.
CONTINUOUS GRAPHS

When a set of data is not confined to separate points and instead consists of connected points, the data is called **continuous**. “John’s Giant Redwood,” problem 3-11, is an example of a continuous situation, because even though the table focuses on integer values of years (1, 2, 3, etc.), the tree still grows between these values of time. Therefore, the tree has a height at any non-negative value of time (such as 1.1 years after it is planted).

When data for a continuous relationship are graphed, the points are connected to show that the relationship also holds true for all points between the table values. See the example of a continuous graph above.

Note: In this course, tile patterns will represent elements of continuous relationships and will be graphed with a continuous line or curve.

PARABOLAS

One kind of graph you will study in this class is called a **parabola**. Two examples of parabolas are graphed at right. Note that parabolas are smooth “U” shapes, not pointy “V” shapes.

The point where a parabola turns (the highest or lowest point) is called the **vertex**.
INDEPENDENT AND DEPENDENT VARIABLES

When one quantity (such as the height of a redwood tree) depends on another (such as the number of years after the tree was planted), it is called a dependent variable. That means its value is determined by the value of another variable. The dependent variable is usually graphed on the y-axis.

If a quantity, such as time, does not depend on another variable, it is referred to as the independent variable, which is graphed on the x-axis.

For example, in problem 3-46, you compared the amount of a dinner bill with the amount of a tip. In this case, the tip depends on the amount of the dinner bill. Therefore, the tip is the dependent variable, while the dinner bill is the independent variable.

COMPLETE GRAPH

A complete graph has the following components:

- x-axis and y-axis labeled, clearly showing the scale.
- Equation of the graph near the line or curve.
- Line or curve extended as far as possible on the graph.
- Coordinates of special points stated in (x, y) format.

Tables can be formatted horizontally, like the one at right, or vertically, as shown below.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-2</td>
</tr>
<tr>
<td>4</td>
<td>-4</td>
</tr>
</tbody>
</table>

Throughout this course, you will continue to graph lines and other curves. Be sure to label your graphs appropriately.
CIRCLE VOCABULARY, CIRCUMFERENCE, AND AREA

The **radius** of a circle is a line segment from its center to any point on the circle. The term is also used for the length of these segments. More than one radius are called **radii**.

A **chord** of a circle is a line segment joining any two points on a circle.

A **diameter** of a circle is a chord that goes through its center. The term is also used for the length of these chords. The length of a diameter is twice the length of a radius.

The **circumference** (C) of a circle is its perimeter, or the “distance around” the circle.

The number π (read “pi”) is the ratio of the circumference of a circle to its diameter. That is, \( \pi = \frac{\text{circumference}}{\text{diameter}} \). This definition is also used as a way of computing the circumference of a circle if you know the diameter, as in the formula \( C = \pi d \) where \( C \) is the circumference and \( d \) is the diameter. Since the diameter is twice the radius \( (d = 2r) \), the formula for the circumference of a circle using its radius is \( C = \pi (2r) \) or \( C = 2\pi \cdot r \).

The first few digits of \( \pi \) are 3.141592.

To find the **area** (A) of a circle when given its radius \( (r) \), square the radius and multiply by \( \pi \). This formula can be written as \( A = r^2 \cdot \pi \). Another way the area formula is often written is \( A = \pi \cdot r^2 \).
SOLVING A LINEAR EQUATION

When solving an equation like the one shown below, several important strategies are involved.

- **Simplify.** Combine like terms and “make zeros” on each side of the equation whenever possible.
- **Keep equations balanced.** The equal sign in an equation indicates that the expressions on the left and right are balanced. Anything done to the equation must keep that balance.
- **Get x alone.** Isolate the variable on one side of the equation and the constants on the other.
- **Undo operations.** Use the fact that addition is the opposite of subtraction and that multiplication is the opposite of division to solve for x. For example, in the equation $2x = -8$, since the 2 and x are multiplied, dividing both sides by 2 will get x alone.

SOLUTIONS TO AN EQUATION WITH ONE VARIABLE

A **solution** to an equation gives a value of the variable that makes the equation true. For example, when 5 is substituted for x in the equation at right, both sides of the equation are equal. So $x = 5$ is a solution to this equation.

An equation can have more than one solution, or it may have no solution. Consider the examples at right.

Notice that no matter what the value of x is, the left side of the first equation will never equal the right side. Therefore, you say that $x + 2 = x + 6$ has no solution.

However, in the equation $x - 3 = x - 3$, no matter what value x has, the equation will always be true. All numbers can make $x - 3 = x - 3$ true. Therefore, you say the solution for the equation $x - 3 = x - 3$ is all numbers.
**THE DISTRIBUTIVE PROPERTY**

The **Distributive Property** states that for any three terms \( a, b, \) and \( c \):

\[
a(b + c) = ab + ac
\]

That is, when \( a \) multiplies a group of terms, such as \((b + c)\), it multiplies each term of the group. For example, when multiplying \(2(x + 4)\), the 2 multiplies both the \( x \) and the 4. This can be represented with algebra tiles, as shown below.

\[
2(x + 4) = 2 \cdot x + 2 \cdot 4 = 2x + 8
\]

The 2 multiplies each term.