SLOPE is a number that indicates the steepness (or flatness) of a line, as well as its direction (up or down) left to right.

SLOPE is determined by the ratio: \( \frac{\text{vertical change}}{\text{horizontal change}} \) between any two points on a line.

For lines that go up (from left to right), the sign of the slope is positive. For lines that go down (left to right), the sign of the slope is negative.

Any linear equation written as \( y = mx + b \), where \( m \) and \( b \) are any real numbers, is said to be in SLOPE-INTERCEPT FORM. \( m \) is the SLOPE of the line. \( b \) is the Y-INTERCEPT, that is, the point \((0, b)\) where the line intersects (crosses) the y-axis.

If two lines have the same slope, then they are parallel. Likewise, PARALLEL LINES have the same slope.

Two lines are PERPENDICULAR if the slope of one line is the negative reciprocal of the slope of the other line, that is, \( m \) and \( -\frac{1}{m} \). Note that \( m \cdot \left(-\frac{1}{m}\right) = -1 \).

Examples: 3 and \(-\frac{1}{3}\), \(-\frac{2}{3}\) and \(\frac{3}{2}\), \(\frac{5}{4}\) and \(-\frac{4}{5}\)

Two distinct lines that are not parallel intersect in a single point. See "Solving Linear Systems" to review how to find the point of intersection.

Also see the textbook, pages 205, 258, 291, 298, 301, 307-08, and 314.

**Example 1**

Write the slope of the line containing the points \((-1, 3)\) and \((4, 5)\).

First graph the two points and draw the line through them.

Look for and draw a slope triangle using the two given points.

Write the ratio \( \frac{\text{vertical change in } y}{\text{horizontal change in } x} \) using the legs of the right triangle: \( \frac{2}{5} \).

Assign a positive or negative value to the slope (this one is positive) depending on whether the line goes up (+) or down (−) from left to right.
If the points are inconvenient to graph, use a "Generic Slope Triangle", visualizing where the points lie with respect to each other.

**Example 2**

Graph the linear equation \( y = \frac{4}{7}x + 2 \)

Using \( y = mx + b \), the slope in \( y = \frac{4}{7}x + 2 \) is \( \frac{4}{7} \) and the y-intercept is the point \((0, 2)\). To graph, begin at the y-intercept \((0, 2)\). Remember that slope is \( \frac{\text{vertical change}}{\text{horizontal change}} \), so go up 4 units (since 4 is positive) from \((0, 2)\) and then move right 7 units. This gives a second point on the graph. To create the graph, draw a straight line through the two points.

**Example 3**

A line has a slope of \( \frac{3}{4} \) and passes through \((3, 2)\). What is the equation of the line?

Using \( y = mx + b \), write \( y = \frac{3}{4}x + b \). Since \((3, 2)\) represents a point \((x, y)\) on the line, substitute 3 for \(x\) and 2 for \(y\), \(2 = \frac{3}{4}(3) + b\), and solve for \(b\).

\[
2 = \frac{9}{4} + b \Rightarrow 2 - \frac{9}{4} = b \Rightarrow -\frac{1}{4} = b.
\]

The equation is \( y = \frac{3}{4}x - \frac{1}{4} \).

**Example 4**

Decide whether the two lines at right are parallel, perpendicular, or neither (i.e., intersecting).

First find the slope of each equation. Then compare the slopes.

\[
\begin{align*}
5x - 4y &= -6 \\
-4y &= -5x - 6 \\
y &= \frac{-5x - 6}{-4} \\
y &= \frac{5}{4}x + \frac{3}{2}
\end{align*}
\]

The slope of this line is \( \frac{5}{4} \) .

\[
\begin{align*}
-4x + 5y &= 3 \\
5y &= 4x + 3 \\
y &= \frac{4x + 3}{5} \\
y &= \frac{4}{5}x + \frac{3}{5}
\end{align*}
\]

The slope of this line is \( \frac{4}{5} \) .

These two slopes are not equal, so they are not parallel. The product of the two slopes is 1, not -1, so they are not perpendicular. These two lines are neither parallel nor perpendicular, but do intersect.
Example 5

Find two equations of the line through the given point, one parallel and one perpendicular to the given line: \( y = -\frac{5}{2}x + 5 \) and \((-4, 5)\).

<table>
<thead>
<tr>
<th>For the parallel line, use ( y = mx + b ) with the same slope to write ( y = -\frac{5}{2}x + b ).</th>
<th>For the perpendicular line, use ( y = mx + b ) where ( m ) is the negative reciprocal of the slope of the original equation to write ( y = \frac{2}{5}x + b ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substitute the point ((-4, 5)) for ( x ) and ( y ) and solve for ( b ). [ 5 = -\frac{5}{2}(-4) + b \implies 5 = \frac{20}{2} + b \implies -5 = b ]</td>
<td>Substitute the point ((-4, 5)) and solve for ( b ). [ 5 = \frac{2}{5}(-4) + b \implies \frac{33}{5} = b ]</td>
</tr>
<tr>
<td>Therefore the parallel line through ((-4, 5)) is ( y = -\frac{5}{2}x - 5 ).</td>
<td>Therefore the perpendicular line through ((-4, 5)) is ( y = \frac{2}{5}x + \frac{33}{5} ).</td>
</tr>
</tbody>
</table>

Write the slope of the line containing each pair of points.

1. \((3, 4)\) and \((5, 7)\)  
2. \((5, 2)\) and \((9, 4)\)  
3. \((1, -3)\) and \((-4, 7)\)  
4. \((-2, 1)\) and \((2, -2)\)  
5. \((-2, 3)\) and \((4, 3)\)  
6. \((8, 5)\) and \((3, 5)\)

Use a Generic Slope Triangle to write the slope of the line containing each pair of points:

7. \((51, 40)\) and \((33, 72)\)  
8. \((20, 49)\) and \((54, 90)\)  
9. \((10, -13)\) and \((-61, 20)\)

Identify the \( y \)-intercept in each equation.

10. \( y = \frac{1}{2}x - 2 \)  
11. \( y = -\frac{3}{5}x - \frac{5}{3} \)  
12. \( 3x + 2y = 12 \)  
13. \( x - y = -13 \)  
14. \( 2x - 4y = 12 \)  
15. \( 4y - 2x = 12 \)

Write the equation of the line with:

16. slope = \( \frac{1}{2} \) and passing through \((4, 3)\).  
17. slope = \( \frac{2}{3} \) and passing through \((-3, -2)\).
18. slope = \( -\frac{1}{3} \) and passing through \((4, -1)\).  
19. slope = \(-4\) and passing through \((-3, 5)\).
Determine the slope of each line using the highlighted points.

20. 

21. 

22. 

Using the slope and y-intercept, determine the equation of the line.

23. 

24. 

25. 

26. 

Graph the following linear equations on graph paper.

27. \( y = \frac{1}{2} x + 3 \) 
28. \( y = -\frac{3}{5} x - 1 \) 
29. \( y = 4x \) 
30. \( y = -6x + \frac{1}{2} \) 
31. \( 3x + 2y = 12 \)

State whether each pair of lines is parallel, perpendicular, or intersecting.

32. \( y = 2x - 2 \) and \( y = 2x + 4 \) 
33. \( y = \frac{1}{2} x + 3 \) and \( y = -2x - 4 \) 
34. \( x - y = 2 \) and \( x + y = 3 \) 
35. \( y - x = -1 \) and \( y + x = 3 \) 
36. \( x + 3y = 6 \) and \( y = -\frac{1}{3} x - 3 \) 
37. \( 3x + 2y = 6 \) and \( 2x + 3y = 6 \) 
38. \( 4x = 5y - 3 \) and \( 4y = 5x + 3 \) 
39. \( 3x - 4y = 12 \) and \( 4y = 3x + 7 \)

Find an equation of the line through the given point and parallel to the given line.

40. \( y = 2x - 2 \) and \((-3, 5)\) 
41. \( y = \frac{1}{2} x + 3 \) and \((-4, 2)\) 
42. \( x - y = 2 \) and \((-2, 3)\) 
43. \( y - x = -1 \) and \((-2, 1)\) 
44. \( x + 3y = 6 \) and \((-1, 1)\) 
45. \( 3x + 2y = 6 \) and \((2, -1)\) 
46. \( 4x = 5y - 3 \) and \((1, -1)\) 
47. \( 3x - 4y = 12 \) and \((4, -2)\)
Find an equation of the line through the given point and perpendicular to the given line.

48. \( y = 2x - 2 \) and \((-3, 5)\)  
49. \( y = \frac{1}{2} x + 3 \) and \((-4, 2)\)  
50. \( x - y = 2 \) and \((-2, 3)\)  
51. \( y - x = -1 \) and \((-2, 1)\)  
52. \( x + 3y = 6 \) and \((-1, 1)\)  
53. \( 3x + 2y = 6 \) and \((2, -1)\)  
54. \( 4x = 5y - 3 \) and \((1, -1)\)  
55. \( 3x - 4y = 12 \) and \((4, -2)\)

Write an equation of the line parallel to each line below through the given point.

56.  
57.
Answers

1. \( \frac{3}{2} \)  
2. \( \frac{1}{2} \)  
3. \(-2\)  
4. \( -\frac{3}{4} \)
5. 0  
6. 0  
7. \( -\frac{16}{9} \)  
8. \( \frac{41}{34} \)
9. \( -\frac{33}{71} \)  
10. \(0, -2\)  
11. \(0, -\frac{5}{3}\)  
12. \(0, 6\)
13. \((0, 13)\)  
14. \((0, -3)\)  
15. \((0, 3)\)  
16. \(y = \frac{1}{2}x + 1\)
17. \(y = \frac{2}{3}x\)  
18. \(y = -\frac{1}{3}x + \frac{1}{3}\)  
19. \(y = -4x - 7\)  
20. \(\frac{1}{2}\)
21. \(\frac{3}{4}\)  
22. \(-2\)  
23. \(y = 2x - 2\)  
24. \(y = -x + 2\)
25. \(y = \frac{1}{3}x + 2\)  
26. \(y = -2x + 4\)  
27. line with slope \(\frac{1}{2}\) and y-intercept \((0, 3)\)
28. line with slope \(-\frac{3}{5}\) and y-intercept \((0, -1)\)  
29. line with slope \(4\) and y-intercept \((0, 0)\)
30. line with slope \(-6\) and y-intercept \(\left(0, \frac{1}{2}\right)\)  
31. line with slope \(-\frac{3}{2}\) and y-intercept \((0, 6)\)
32. parallel  
33. perpendicular  
34. perpendicular  
35. perpendicular
36. parallel  
37. intersecting  
38. intersecting  
39. parallel
40. \(y = 2x + 11\)  
41. \(y = \frac{1}{2}x + 4\)  
42. \(y = x + 5\)  
43. \(y = x + 3\)
44. \(y = -\frac{1}{3}x + \frac{2}{3}\)  
45. \(y = -\frac{3}{2}x + 2\)  
46. \(y = \frac{4}{5}x - \frac{9}{5}\)  
47. \(y = \frac{3}{4}x - 5\)
48. \(y = -\frac{1}{2}x + \frac{7}{2}\)  
49. \(y = -2x - 6\)  
50. \(y = -x + 1\)  
51. \(y = -x - 1\)
52. \(y = 3x + 4\)  
53. \(y = \frac{2}{3}x - \frac{7}{3}\)  
54. \(y = -\frac{5}{4}x + \frac{1}{4}\)  
55. \(y = -\frac{4}{3}x + \frac{10}{3}\)
56. \(y = 3x + 11\)  
57. \(y = -\frac{1}{2}x + \frac{15}{2}\)

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