ZERO PRODUCT PROPERTY AND QUADRATICS #16

If \( a \cdot b = 0 \), then either \( a = 0 \) or \( b = 0 \).

Note that this property states that at least one of the factors MUST be zero. It is also possible that all of the factors are zero. This simple statement gives us a powerful result which is most often used with equations involving the products of binomials. For example, solve \((x + 5)(x - 2) = 0\).

By the Zero Product Property, since \((x + 5)(x - 2) = 0\), either \(x + 5 = 0\) or \(x - 2 = 0\). Thus, \(x = -5\) or \(x = 2\).

The Zero Product Property can be used to find where a quadratic equation crosses the x-axis. These points are the x-intercepts. In the example above, they would be (-5, 0) and (2, 0). Also see the textbook, pages 346-49.

Here are two more examples. Solve each quadratic equation and check each solution.

**Example 1**

\[(x + 4)(x - 7) = 0\]

By the Zero Product Property, either \(x + 4 = 0\) or \(x - 7 = 0\)

Solving, \(x = -4\) or \(x = 7\).

Checking,

\((-4 + 4)(-4 - 7) = 0\)
\((0)(-11) = 0\sqrt{}\)
\((7 + 4)(7 - 7) = 0\)
\((11)(0) = 0\sqrt{}\)

**Example 2**

\[x^2 + 3x - 10 = 0\]

First factor \(x^2 + 3x - 10 = 0\) into \((x + 5)(x - 2) = 0\)

then \(x + 5 = 0\) or \(x - 2 = 0\), so \(x = -5\) or \(x = 2\)

Checking,

\((-5 + 5)(-5 - 2) = 0\)
\((0)(-7) = 0\sqrt{}\)
\((2 + 5)(2 - 2) = 0\)
\((7)(0) = 0\sqrt{}\)
Solve each of the following quadratic equations.

1. \((x + 7)(x + 1) = 0\)
2. \((x + 2)(x + 3) = 0\)
3. \(x(x - 2) = 0\)
4. \(x(x - 7) = 0\)
5. \((3x - 3)(4x + 2) = 0\)
6. \((2x + 5)(4x - 3) = 0\)
7. \(x^2 + 4x + 3 = 0\)
8. \(x^2 + 6x + 5 = 0\)
9. \(x^2 - 6x + 8 = 0\)
10. \(x^2 - 8x + 15 = 0\)
11. \(x^2 + x = 6\)
12. \(x^2 - x = 6\)
13. \(x^2 - 10x = -16\)
14. \(x^2 - 11x = -28\)

Without graphing, find where each parabola crosses the \(x\)-axis.

15. \(y = x^2 - 2x - 3\)
16. \(y = x^2 + 2x - 8\)
17. \(y = x^2 - x - 30\)
18. \(y = x^2 + 4x - 5\)
19. \(x^2 + 4x = 5 + y\)
20. \(x^2 - 3x = 10 + y\)

**Answers**

1. \(x = -7\) and \(x = -1\)
2. \(x = -2\) and \(x = -3\)
3. \(x = 0\) and \(x = 2\)
4. \(x = 0\) and \(x = 7\)
5. \(x = 1\) and \(x = -\frac{1}{2}\)
6. \(x = \frac{-5}{2}\) and \(x = \frac{3}{4}\)
7. \(x = -1\) and \(x = -3\)
8. \(x = -1\) and \(x = -5\)
9. \(x = 4\) and \(x = 2\)
10. \(x = 5\) and \(x = 3\)
11. \(x = -3\) and \(x = 2\)
12. \(x = 3\) and \(x = -2\)
13. \(x = 2\) and \(x = 8\)
14. \(x = 4\) and \(x = 7\)
15. \((-1, 0)\) and \((3, 0)\)
16. \((-4, 0)\) and \((2, 0)\)
17. \((6, 0)\) and \((-5, 0)\)
18. \((-5, 0)\) and \((1, 0)\)
19. \((1, 0)\) and \((-5, 0)\)
20. \((5, 0)\) and \((-2, 0)\)