ABSOLUTE VALUE INEQUALITIES

To solve absolute value inequalities, rewrite the inequality without absolute value according to the two patterns below and solve in the usual ways.

1) \(|x| \leq a\) is equivalent to \(-a \leq x \leq a\)
   (all values of \(x\) that have a distance from 0 of less than or equal to “\(a\)”).

2) \(|x| \geq a\) is equivalent to \(x \geq a\) or \(x \leq -a\)
   (all values of \(x\) that have a distance from 0 of greater than or equal to “\(a\)”).

For more complicated inequalities, follow the same patterns and simplify. Less than (<) and greater than (>) is solved in the same way as patterns one and two above using the appropriate sign. Also see the textbook, page 432.

Example 1
Solve: \(|x| \leq 4\)
Using pattern 1: \(-4 \leq x \leq 4\)
   (all numbers between -4 and 4, inclusive).

Example 2
Solve: \(|2y| > 4\)
Using pattern 2: \(2y > 4\) or \(2y < -4\).
   Simplify: \(y > 2\) or \(y < -2\)
   (all numbers above 2 or below -2)

Example 3
Solve: \(|2y + 1| < 5\)
Using pattern 1: \(-5 < 2y + 1 < 5\)
Simplify: \(-6 < 2y < 4\)
   \(-3 < y < 2\)
   (all numbers between -3 and 2)

Example 4
Solve: \(|3x - 2| + 1 > 7\)
Simplify: \(|3x - 2| > 6\)
Using pattern 2: \(3x - 2 > 6\) or \(3x - 2 < -6\)
Simplify: \(3x > 8\) or \(3x < -4\)
   \(x > \frac{8}{3}\) or \(x < -\frac{4}{3}\)
   (all numbers above \(\frac{8}{3}\) or below \(-\frac{4}{3}\))
Solve each absolute value inequality.

1. $|x| \leq 3$
2. $|y| > 4$
3. $|2x| < 8$
4. $|3y| \geq 9$
5. $|x - 1| < 7$
6. $|2y + 1| \geq 11$
7. $|3m - 1| \leq 5$
8. $|3m| + 7 < 10$
9. $|1 + 2y| > 10$
10. $\left| \frac{x}{2} - 3 \right| \leq 12$
11. $2|x| + 3 \leq 10$
12. $8|2x + 1| - 1 < 63$

Answers

1. $-3 \leq x \leq 3$
2. $y > 4$ or $y < -4$
3. $-4 < x < 4$
4. $y > 3$ or $y < -3$
5. $-6 < x < 8$
6. $y \geq 5$ or $y \leq -6$
7. $\frac{-3}{2} \leq m \leq 2$
8. $-1 < m < 1$
9. $y > \frac{9}{2}$ or $y < -\frac{11}{2}$
10. $-18 \leq x \leq 30$
11. $\frac{-7}{2} \leq x \leq \frac{7}{2}$
12. $\frac{-9}{2} < x < \frac{7}{2}$