RIGID TRANSFORMATIONS

A **RIGID TRANSFORMATION** is a transformation of a figure that preserves size and shape. The three kinds of rigid transformations studied are translation (slide), reflection (flip), and rotation (turn). These transformations may also be combined together. The original figure is called the pre-image and the new figure is called the image. For $\triangle ABC$ it is common to say that the original points $A$, $B$, and $C$ are mapped to the new points $A'$, $B'$, and $C'$ respectively. For more information about rigid transformations see the Math Notes boxes on pages 33 and 38.

**Example 1**
Translate (slide) $\triangle ABC$ right six units and up three units. Give the coordinates of the image triangle.

The original vertices are $A(-5, -2)$, $B(-3, 1)$, and $C(0, -5)$. The new vertices are $A'(1, 1)$, $B'(3, 4)$, and $C'(6, -2)$. Notice that each point $(x, y)$ is mapped to $(x + 6, y + 3)$.

**Example 2**
Reflect (flip) $\triangle ABC$ with coordinates $A(5, 2)$, $B(2, 4)$, and $C(4, 6)$ across the $y$-axis to get $\triangle A'B'C'$. The key is that the reflection is the same distance from the axis as the original figure. The new points are $A'(-5, 2)$, $B'(-2, 4)$, and $C'(-4, 6)$. Notice that in reflecting across the $y$-axis, each point $(x, y)$ is mapped to the point $(-x, y)$.

If you reflect $\triangle ABC$ across the $x$-axis to get $\triangle PQR$, then the new points are $P(5, -2)$, $Q(2, -4)$, and $R(4, -6)$. In this case, reflecting across the $x$-axis, each point $(x, y)$ is mapped to the point $(x, -y)$.

**Example 3**
Rotate (turn) $\triangle ABC$ with coordinates $A(2, 0)$, $B(6, 0)$, and $C(3, 4)$ $90^\circ$ counterclockwise about the origin $(0, 0)$ to get $\triangle A'B'C'$ with coordinates $A'(0, 2)$, $B'(0, 6)$, and $C'(-4, 3)$. Notice that this $90^\circ$ counterclockwise rotation about the origin maps each point $(x, y)$ to the point $(-y, x)$.

Rotating another $90^\circ$ ($180^\circ$ from the starting location) yields $\triangle A''B''C''$ with coordinates $A''(-2, 0)$, $B''(-6, 0)$, and $C''(-3, -4)$. This $180^\circ$ counterclockwise rotation about the origin maps each point $(x, y)$ to the point $(-x, -y)$. Similarly a $270^\circ$ counterclockwise or $90^\circ$ clockwise rotation about the origin maps each point $(x, y)$ to the point $(y, -x)$. 

GEOMETRY Connections
For the following problems, refer to the figures below:

Tell the new coordinates after each transformation.

1. Slide figure A left 2 units and down 3 units.
2. Slide figure B right 3 units and down 5 units.
3. Slide figure C left 1 unit and up 2 units.
4. Flip figure A across the x-axis.
5. Flip figure B across the x-axis.
6. Flip figure C across the x-axis.
7. Flip figure A across the y-axis.
8. Flip figure B across the y-axis.
9. Flip figure C across the y-axis.
10. Rotate figure A 90° counterclockwise about the origin.
11. Rotate figure B 90° counterclockwise about the origin.
12. Rotate figure C 90° counterclockwise about the origin.
13. Rotate figure A 180° counterclockwise about the origin.
14. Rotate figure C 180° counterclockwise about the origin.
15. Rotate figure B 270° clockwise about the origin.
16. Rotate figure C 90° clockwise about the origin.

**Answers (given in order A', B', C')**

1. (-1, -3) (1, 2) (3, -1) 2. (-2, -3) (2, -3) (3, 0)
3. (-5, 4) (3, 4) (-3, -1) 4. (1, 0) (3, -4) (5, -2)
5. (-5, -2) (-1, -2) (0, -5) 6. (-4, -2) (4, -2) (-2, 3)
7. (-1, 0) (-3, 4) (-5, 2) 8. (5, 2) (1, 2) (0, 5)
9. (4, 2) (-4, 2) (2, -3) 10. (0, 1) (-4, 3) (-2, 5)
11. (-2, -5) (-5, 0) (-2, -1) 12. (-2, -4) (-2, 4) (3, -2)
13. (-1, 0) (-3, -4) (-5, -2) 14. (4, -2) (-4, -2) (2, 3)
15. (2, 5) (2, 1) (5, 0) 16. (2, 4) (2, -4) (-3, 2)