PROOF

A proof convinces an audience that a conjecture is true for ALL cases (situations) that fit the conditions of the conjecture. For example, "If a polygon is a triangle on a flat surface, then the sum of the measures of the angles is 180°." Because we proved this conjecture in chapter two, it is always true. There are many formats that may be used to write a proof. This course explores three of them, namely, paragraph, flow chart, and two-column.

Example
If \( \overline{BD} \) is a perpendicular bisector of \( \overline{AC} \), prove that \( \triangle ABC \) isosceles.

**Paragraph proof**

To prove that \( \triangle ABC \) is isosceles, show that \( \overline{BA} \cong \overline{BC} \). We can do this by showing that the two segments are corresponding parts of congruent triangles.

Since \( \overline{BD} \) is perpendicular to \( \overline{AC} \), \( m\angle BDA = m\angle BDC = 90° \).

Since \( \overline{BD} \) bisects \( \overline{AC} \), \( \overline{AD} \cong \overline{CD} \). With \( \overline{BD} \cong \overline{BD} \) (reflexive property), \( \triangle ADB \cong \triangle CDB \) by SAS.

Finally, \( \overline{BA} \cong \overline{BC} \) because corresponding parts of congruent triangles are congruent. Therefore, \( \triangle ABC \) must be isosceles since two of the three sides are congruent.

**Flow chart proof**

Given: \( \overline{BD} \) is the perpendicular bisector of \( \overline{AC} \)

\[ \angle ADB \cong \angle BDC \]

\[ \overline{AD} \cong \overline{CD} \]

\[ \triangle ABD \cong \triangle CBD \]

\[ \angle ADB \cong \angle BDC \]

\[ \triangle ABC \text{ is isosceles} \]

**Two-Column Proof**

Given: \( \overline{BD} \) is a bisector of \( \overline{AC} \).

\( \overline{BD} \) is perpendicular to \( \overline{AC} \).

Prove: \( \triangle ABC \) is isosceles

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \overline{BD} ) bisects ( \overline{AC} ).</td>
<td>Given</td>
</tr>
<tr>
<td>( \overline{BD} \perp \overline{AC} )</td>
<td>Given</td>
</tr>
<tr>
<td>( \overline{AD} \cong \overline{CD} )</td>
<td>Def. of bisector</td>
</tr>
<tr>
<td>( \angle ADB ) and ( \angle BDC ) are right angles</td>
<td>Def. of perpendicular</td>
</tr>
<tr>
<td>( \angle ADB \cong \angle BDC )</td>
<td>All right angles are ( \cong ).</td>
</tr>
<tr>
<td>( \overline{BD} \cong \overline{BD} )</td>
<td>Reflexive property</td>
</tr>
<tr>
<td>( \triangle ABD \cong \triangle CBD )</td>
<td>S.A.S.</td>
</tr>
<tr>
<td>( \overline{AB} \cong \overline{CB} )</td>
<td>( \cong )s have ( \cong ) parts</td>
</tr>
<tr>
<td>( \therefore \triangle ABC \text{ is isosceles} )</td>
<td>Def. of isosceles</td>
</tr>
</tbody>
</table>
In each diagram below, are any triangles congruent? If so, prove it. (Note: It is good practice to try different methods for writing your proofs.)

1. 

2. 

3. 

4. 

5. 

6. 

Complete a proof for each problem below in the style of your choice.

7. Given: TR and MN bisect each other. Prove: \(\triangle NTP \cong \triangle MRP\)

8. Given: CD bisects \(\angle ACB\); \(\angle 1 \equiv \angle 2\). Prove: \(\triangle CDA \cong \triangle CDB\)

9. Given: \(AB \parallel CD\), \(\angle B \equiv \angle D\), \(AB \equiv CD\). Prove: \(\triangle ABF \cong \triangle CED\)

10. Given: \(PG \equiv SG\), \(TP \equiv TS\). Prove: \(\triangle TPG \cong \triangle TSG\)

11. Given: \(OE \perp MP\), \(OE\) bisects \(\angle MOP\). Prove: \(\triangle MOE \cong \triangle POE\)

12. Given: \(AD \parallel BC\), \(DC \parallel BA\). Prove: \(\triangle ADB \cong \triangle CBD\)
13. Given: $\overline{AC}$ bisects $\overline{DE}$, $\angle A \equiv \angle C$
Prove: $\triangle ADB \equiv \triangle CEB$

14. Given: $\overline{PQ} \perp \overline{RS}$, $\angle R \equiv \angle S$
Prove: $\triangle PQR \equiv \triangle PQS$

15. Given: $\angle S \equiv \angle R$, $\overline{PQ}$ bisects $\angle SQR$
Prove: $\triangle SPQ \equiv \triangle RPQ$

16. Given: $\overline{TU} \equiv \overline{GY}$, $\overline{KY} \parallel \overline{HU}$, $\overline{KT} \perp \overline{TG}$, $\overline{HG} \perp \overline{TG}$
Prove: $\angle K \equiv \angle H$

17. Given: $\overline{MQ} \parallel \overline{WL}$, $\overline{MQ} \equiv \overline{WL}$
Prove: $\overline{ML} \parallel \overline{WQ}$

18. Is $\triangle BCD \equiv \triangle EDC$? Prove it!

19. Is $\overline{AB} \equiv \overline{DC}$? Prove it!

20. Is $\overline{AB} \equiv \overline{ED}$? Prove it!

Answers
1. Yes
2. Yes
3. Yes

![Diagram of angle bisector showing \( \angle BCD \equiv \angle BCA \) and \( \Delta ABC \equiv \Delta DBC \).]

4. Yes

![Diagram showing \( \Delta ABC \equiv \Delta CDA \).]

5. Not necessarily. Counterexample:

![Counterexample diagram showing non-congruent triangles.]

6. Yes

![Diagram showing \( \Delta ABC \equiv \Delta DEF \).]

7. \( \overline{NP} \equiv \overline{MP} \) and \( \overline{TP} \equiv \overline{RP} \) by definition of bisector. \( \angle NPT \equiv \angle MPR \) because vertical angles are equal. So, \( \Delta NTP \equiv \Delta MRP \) by SAS.

8. \( \angle ACD \equiv \angle BCD \) by definition of angle bisector. \( \overline{CD} \equiv \overline{CD} \) by reflexive so \( \Delta CDA \equiv \Delta CDB \) by ASA.

9. \( \angle A \equiv \angle C \) since alternate interior angles of parallel lines congruent so \( \Delta ABF \equiv \Delta CED \) by ASA.

10. TG \( \equiv \) TG by reflexive so \( \Delta TPG \equiv \Delta TSG \) by SSS.

11. \( \angle MEO \equiv \angle PEO \) because perpendicular lines form \( \equiv \) right angles \( \angle MOE \equiv \angle POE \) by angle bisector and \( \overline{OE} \equiv \overline{OE} \) by reflexive. So, \( \Delta MOE \equiv \Delta POE \) by ASA.

12. \( \angle CDB \equiv \angle ABD \) and \( \angle ADB \equiv \angle CBD \) since parallel lines give congruent alternate interior angles. \( \overline{DB} \equiv \overline{DB} \) by reflexive so \( \Delta ADB \equiv \Delta CBD \) by ASA.

13. \( \overline{DB} \equiv \overline{EB} \) by definition of bisector. \( \angle DBA \equiv \angle EBC \) since vertical angles are congruent. So \( \Delta ADB \equiv \Delta CEB \) by AAS.

14. \( \angle RQP \equiv \angle SQP \) since perpendicular lines form congruent right angles. \( \overline{PQ} \equiv \overline{PQ} \) by reflexive so \( \Delta PQR \equiv \Delta PQS \) by AAS.

15. \( \angle SQP \equiv \angle RQP \) by angle bisector and \( \overline{PQ} \equiv \overline{PQ} \) by reflexive, so \( \Delta SPQ \equiv \Delta RPQ \) by AAS.

16. \( \angle KYT \equiv \angle HUG \) because parallel lines form congruent alternate exterior angles. \( \overline{TY} \equiv \overline{YU} + \overline{GU} \) so \( \overline{TY} \equiv \overline{GU} \) by subtraction. \( \angle T \equiv \angle G \) since perpendicular lines form congruent right angles. So \( \Delta KTY \equiv \Delta HGU \) by ASA. Therefore, \( \angle K \equiv \angle H \) since \( \equiv \) triangles have congruent parts.

17. \( \angle MQL \equiv \angle WLQ \) since parallel lines form congruent alternate interior angles. \( \overline{QL} \equiv \overline{QL} \) by reflexive so \( \Delta MQL \equiv \Delta WLQ \) by SAS so \( \angle WQL \equiv \angle MLQ \) since congruent triangles have congruent parts. So \( \overline{ML} \parallel \overline{WQ} \) since congruent alternate interior angles are formed by parallel lines.

18. Yes

![Diagram showing \( \overline{DB} \equiv \overline{CE} \).]


20. Not necessarily.